Student Perspectives on Summer School Versus Term-Time for Undergraduate Mathematics

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Abstract

Earlier studies at The University of Sydney indicate that students undertaking certain first year mathematics units in intensive mode of delivery (IMD) achieved superior learning outcomes compared to those completing the same units during the semester. The aim of this study is to survey students that took any undergraduate mathematics units offered in IMD over the period 2009-2016, asking them to compare summer school with semester learning environments. While data suggest that the learning environment is overwhelmingly in favour of summer school, there are features of both modes that appear to be successful. This leads to a flow-diagram, akin to Biggs' Presage-Process-Product (3P) model, emphasising presage and temporality.

Introduction

Easdown, Ougrinovskaia, Saunders, Warren, Ancev, Bishop and Mansfield (2009), and later Easdown, Papadopoulos and Zheng (2019), showed that students who failed certain first year mathematics units had superior learning outcomes and overall course satisfaction by completing them in intensive mode of delivery (IMD) rather than during semester. Both studies investigated, more broadly, reasons and influences for the success or otherwise of IMDs. The second study interpreted findings in terms of the SOLO taxonomy (Biggs & Collis, 1982) and navigation through liminal space (Meyer & Land, 2005; Cousin, 2007). These studies respond to the "call to arms" for research comparing IMD with semester-long modes (Daniel, 2000; Davies, 2006), especially addressing the paucity of IMD research in Australian contexts. The aim of this study is to make a direct comparison between IMD and semester learning environments, from the point of view of the student, and endeavour to tease out the underlying dynamics that lead to successful learning outcomes and course satisfaction. We explore the range and depth of possible effects and influences, which can be highly nuanced, and context driven, using a modification of Biggs' Presage-Process-Product (3P) model. Findings may have implications for providing advice, particularly for students at risk, and for improving learning and teaching practices, in the context of demands for flexibility and novel pathways for completion of degree programs (University of Sydney, 2016, Strategy 5: Transform the learning experience).

Biggs (1993a) abstracts relationships using his Presage-Process-Product (3P) model, a refinement of models introduced by Dunkin and Biddle (1974) (Figure 1). This captures relationships between characteristics of the learner and learning context (Presage), the activity of learning and interactions with teaching (Process) and learning outcomes (Product). As well as a general flow from left to right, students' experiences exist within a dynamical system with

feedback, possibly leading to equilibrium. Modifying any component may propagate effects throughout, leading to different equilibria. Some systems promote surface approaches to learning, with poorer outcomes, whilst others promote deep approaches, with higher quality outcomes.

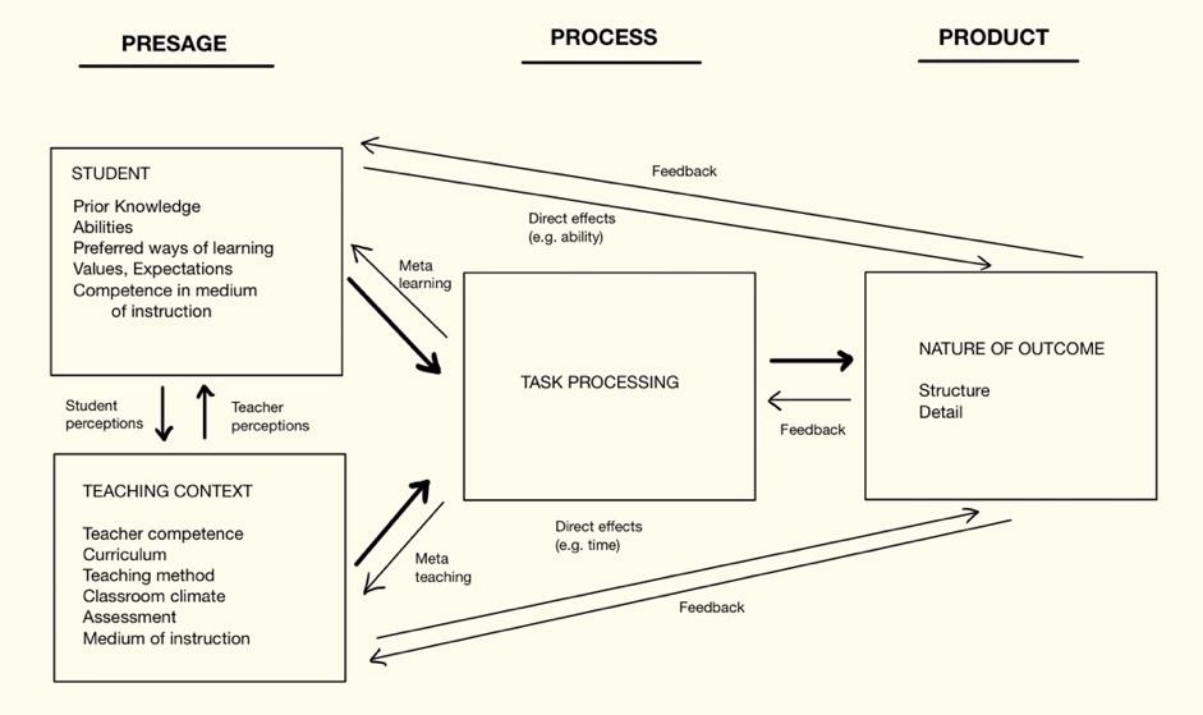

Figure 1. The Biggs 3P Model

Background and methodology

The University of Sydney, in the period 2009-2016, offered nine mathematics units of study in two ways; semester-long and The Sydney Summer School (IMD) (see Appendix 4 in Supplementary Material for background and details). We expand the scope of earlier studies, surveying students taking any mathematics units of study offered at Summer School over this period. In 2017, invitations were sent to students who had participated in Summer School. These invitations included surveys using SurveyMonkey, as part of the doctoral research project of the first author. Of these, 181 responses (7.4%) were received from alumni within the period 2009-2016.

These surveys comprised thirty-six questions (see Appendix 1 in Supplementary Material), probing issues relating to learning, study methods and assessments, both at Summer School and during semester. Thirty questions used a five-point Likert scale, with the opportunity to also provide open-ended responses. Six further questions were open-ended. Likert options were of two types:

- Type 1 (25 questions) – Strongly Disagree, Disagree, Neutral, Agree, Strongly Agree. These were asked for each of Summer School and term-time.
- Type 2 (5 questions) – Term-time (by a large margin), Term-time (by a small margin), Indifferent, Summer School (by a small margin), Summer School (by a large margin).

The histograms (Figures 2-31) suggest that the learning environment was overwhelmingly in favour of the intensive mode of delivery. There are features of both modes that appeared to be successful, so a qualitative coding analysis was applied to over one thousand open-ended comments, teasing out important factors that influence learning and course satisfaction. The following phenomenographic technique was applied:

1. Open-ended responses were split into items and placed into a large pool.
2. Each item was identified with a key word.
3. Key words were divided into groupings.
4. Groupings were amalgamated into categories and sub-categories (see Appendix 2 in Supplementary Material).
5. Relationships were sought, and a mapping made to the Presage, Process and Product phases of a 3P diagram (Figure 32).

Iterations produced seven categories and ten associated sub-categories (see Results below, and Appendix 2 in Supplementary Material). Relationships suggested a flow-diagram in the form of a 3P model (Figure 32), emphasising presage and temporality, much like the "culturally modified" 3P model of Biggs (1996), used to investigate the so-called "paradox of the Chinese learner" (see Appendix 3 in Supplementary Material).

Quantitative results

Numerical values were assigned in a standard way (see Appendix 1 in Supplementary Material) for finding *p*-values. Histograms, means, and medians were produced, and statistical tests of significance were performed. Histograms for Likert Type 1 data use the following key:

Strongly Disagree Disagree Neutral Agree Strongly Agree

The sign test was used against the alternative hypothesis that the Summer School median is greater than the term-time median. For Likert Type 2 data, we test against the alternative hypothesis that the median is greater than zero, in favour of Summer School over term-time. Representative student comments appear in Appendix 4 in Supplementary Material.

Instruction
The *p*-values are less than 10^{-15}.

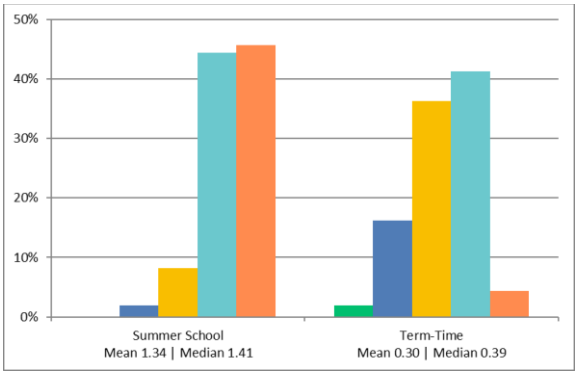

Figure 2 – Overall I was satisfied with the quality of teaching by the teacher(s).

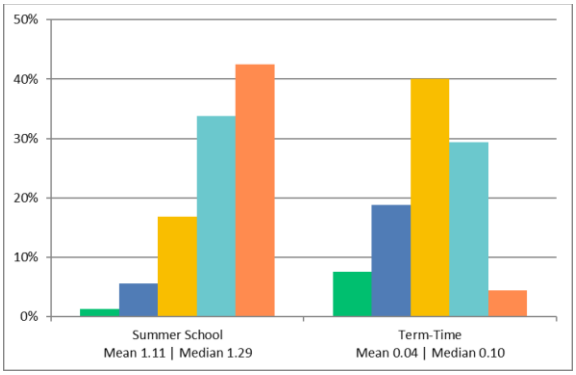

Figure 3 – I had been guided by helpful feedback on my learning.

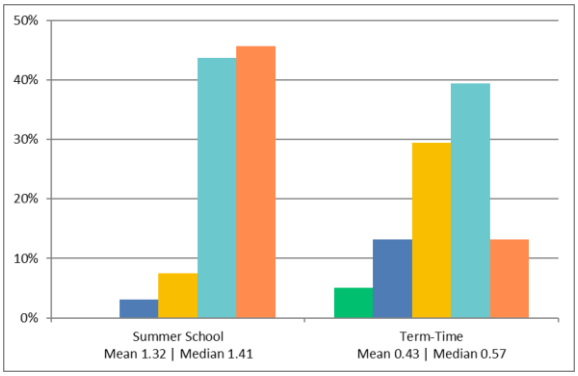

Figure 4 – Staff were responsive to students.

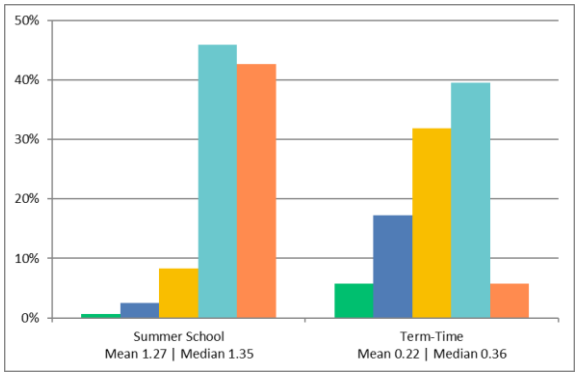

Figure 5 – The lecturers were effective in facilitating my learning.

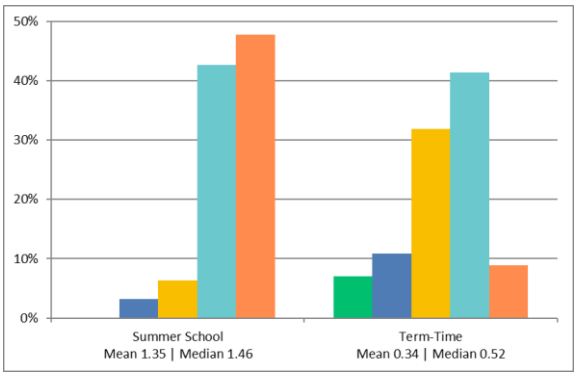

Figure 6 – The tutors were effective in facilitating my learning.

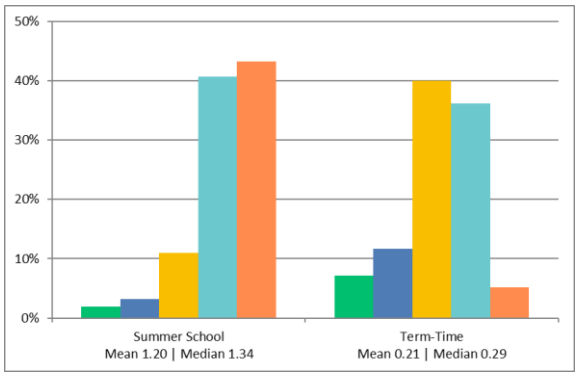

Figure 7 – The feedback in relation to assessment tasks was timely and of high quality.

Learning

The *p*-values are less than 10^{-12}.

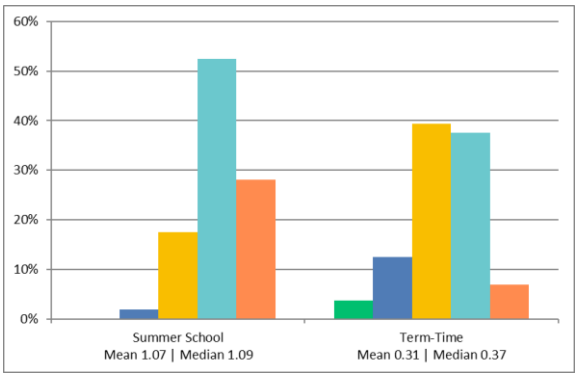

Figure 8 – I developed relevant critical and analytical thinking skills.

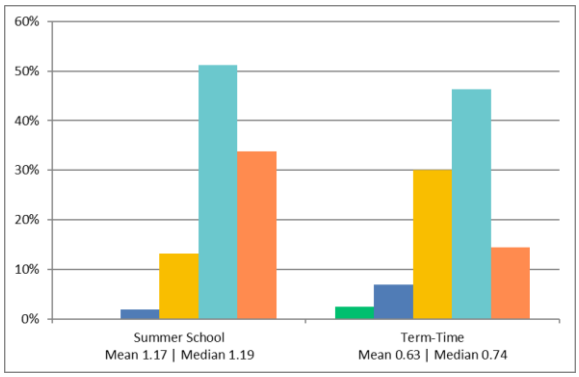

Figure 9 – Learning outcomes were clear to me.

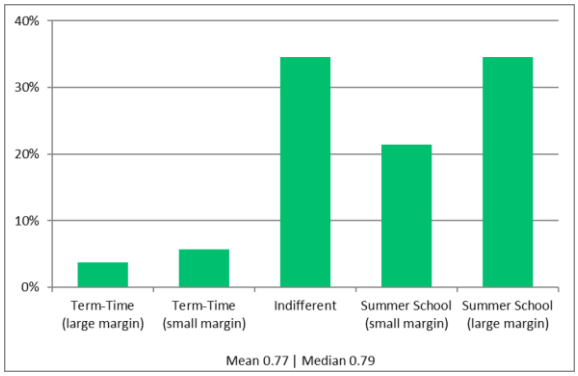

Figure 10 – Which mode of delivery did you find provided you with superior educational and learning outcomes?

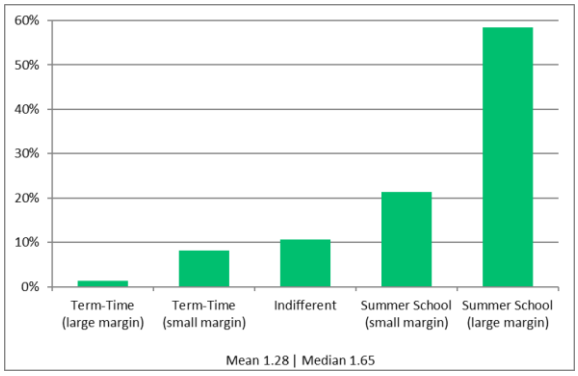

Figure 11 – Which mode of delivery did you find provided you with better compatibility with your own personal style of learning?

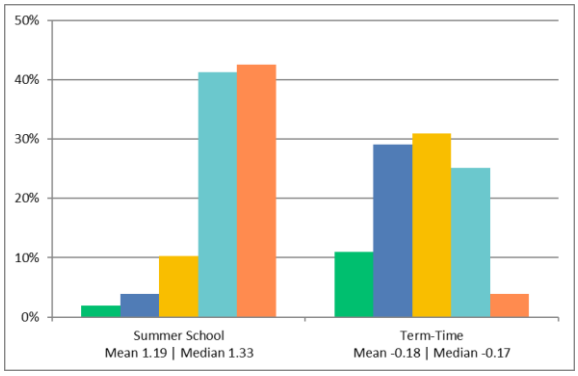

Figure 12 – I was able to focus on study without distraction.

Classes

The *p*-values are less than 10^{-18}.

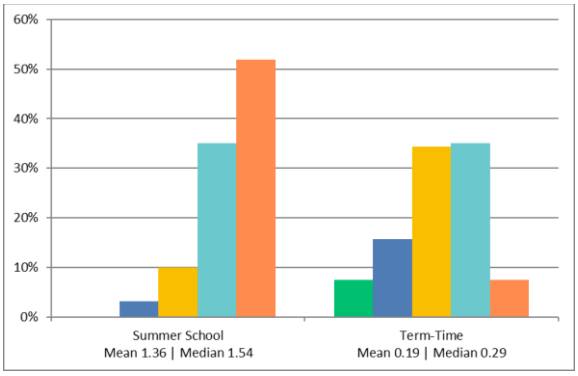

Figure 13 – Tutorials helped me to learn.

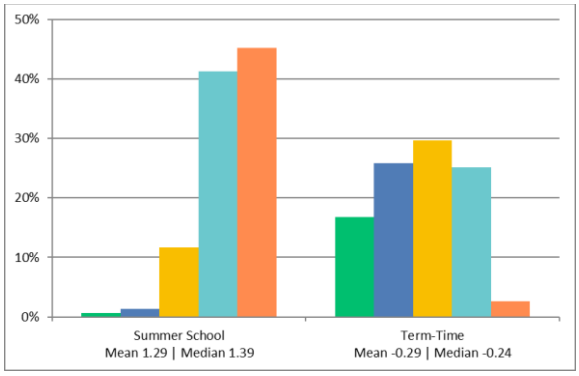

Figure 14 – The lecture class sizes were appropriate for facilitating my learning.

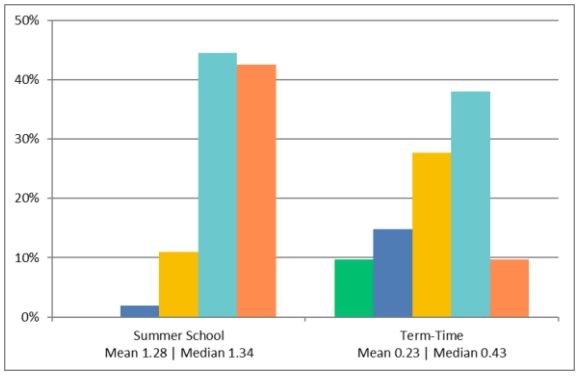

Figure 15 – The tutorial class sizes were appropriate for facilitating my learning.

Motivation

The *p*-value is less than 10^{-23}.

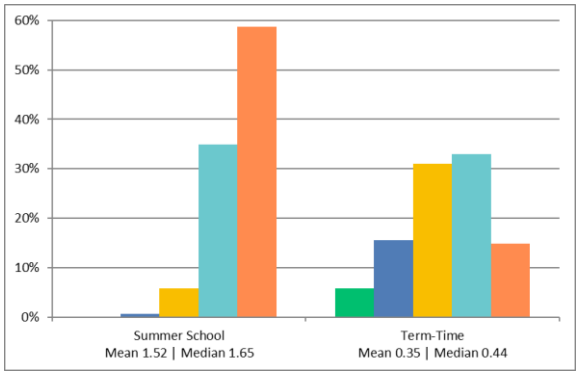

Figure 16 – I was personally motivated to pass or do well.

Pace and Timing

The *p*-values are less than 10^{-11}.

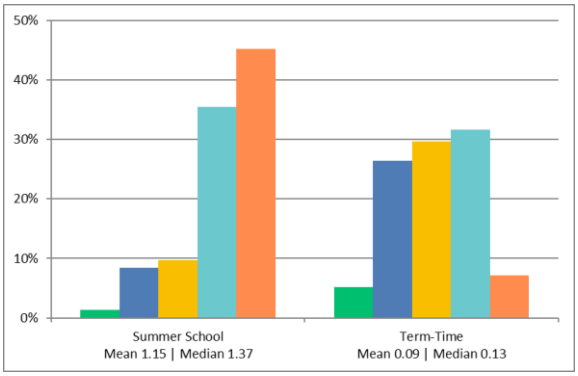

Figure 17 – The pace was beneficial in facilitating my learning.

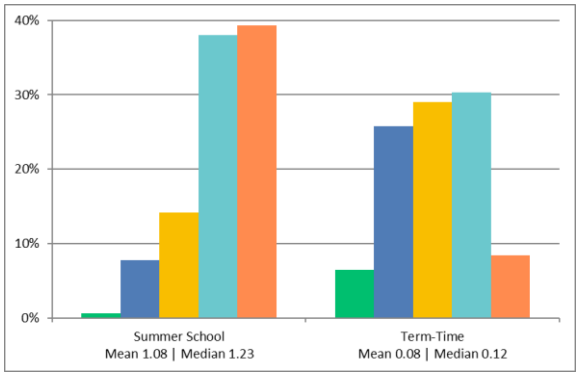

Figure 18 – The timing was beneficial in facilitating my learning.

Enjoyment

The *p*-values are less than 10^{-14}.

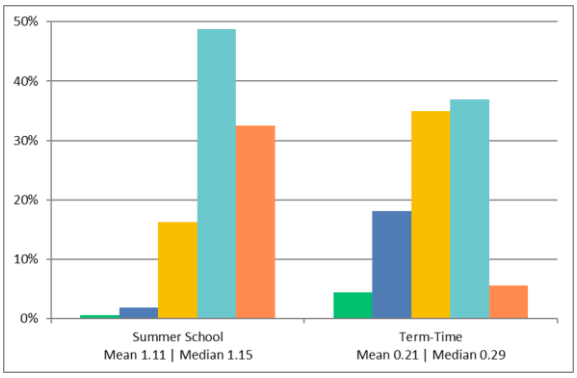

Figure 19 – The work had been intellectually rewarding.

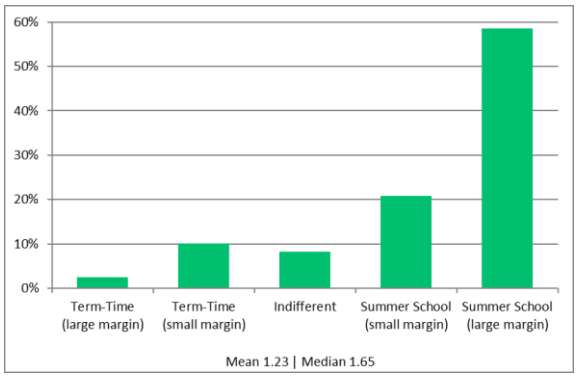

Figure 20 – Which mode of delivery did you find provided you with more enjoyment and satisfaction?

Resources

The *p*-values are less than 10^{-6}.

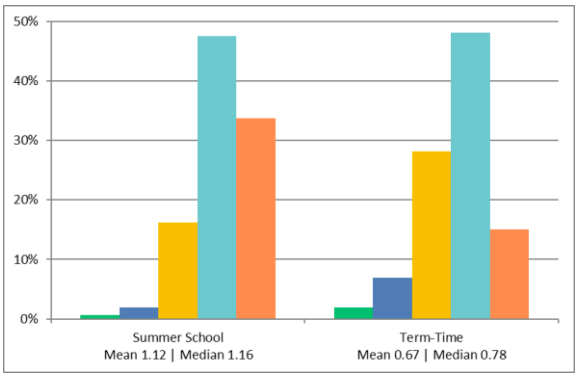

Figure 21 – I had good access to valuable learning resources.

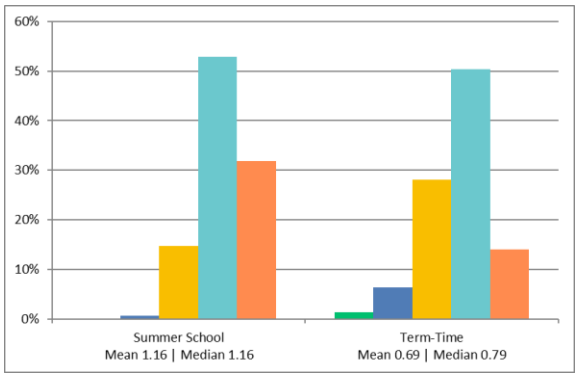

Figure 22 – The unit of study materials were effective in facilitating my learning.

Assessment

The *p*-values are less than 10^{-4}.

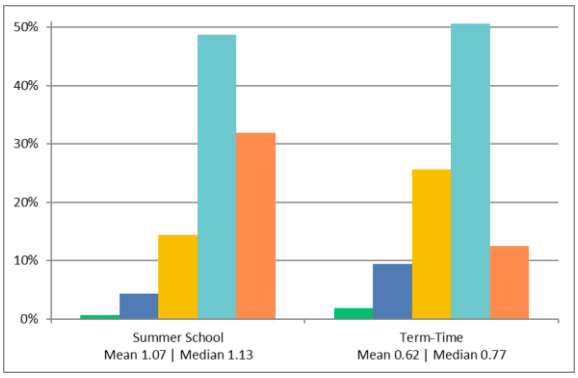

Figure 23 – The assessment tasks challenged me to learn.

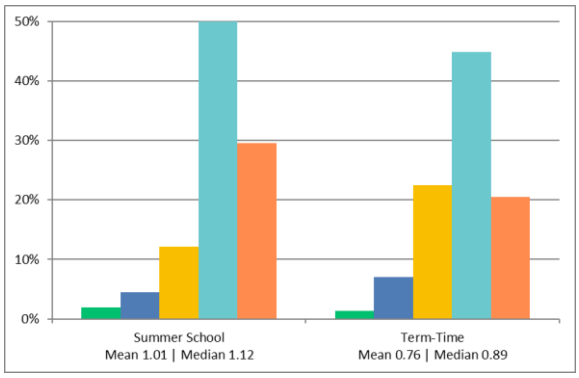

Figure 24 – The exams were effective in testing my knowledge, understanding and aptitude.

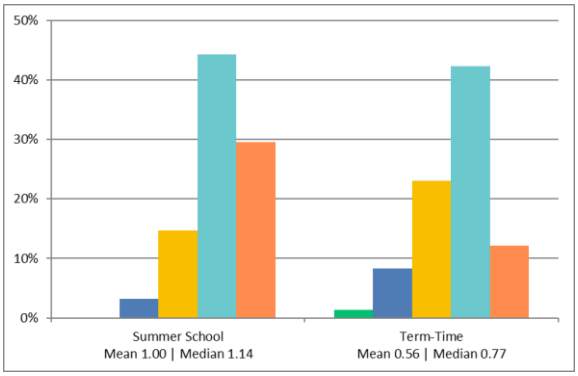

Figure 25 – The quizzes were effective in testing my knowledge, understanding and aptitude.

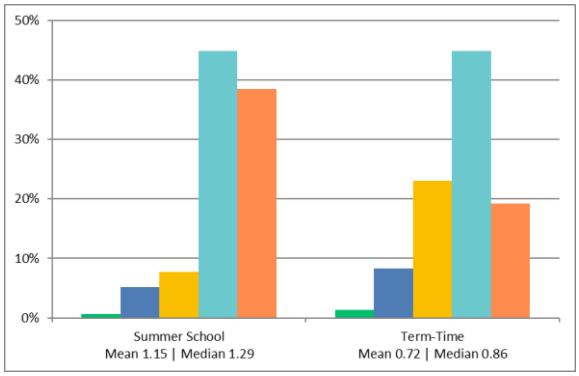

Figure 26 – The assignments were effective in testing my knowledge, understanding and aptitude.

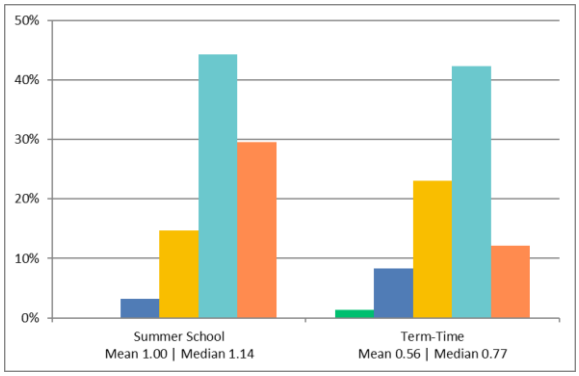

Figure 27 – The homework was effective in testing my knowledge, understanding and aptitude.

Easiness

The *p*-value is less than 10^{-13}.

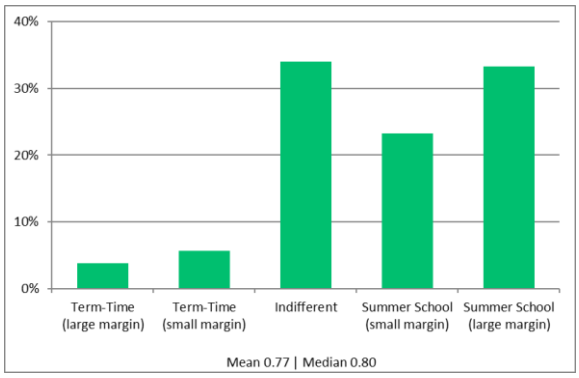

Figure 28 – Which mode of delivery did you find provided you with units of study that were overall easier?

Social

The *p*-values are less than 10^{-3}.

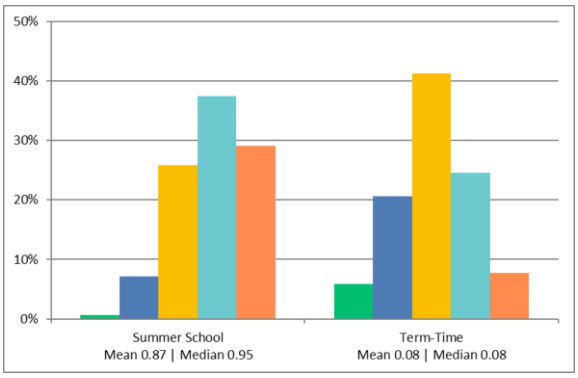

Figure 29 – Social context and interaction within/during scheduled classes was beneficial in facilitating my learning.

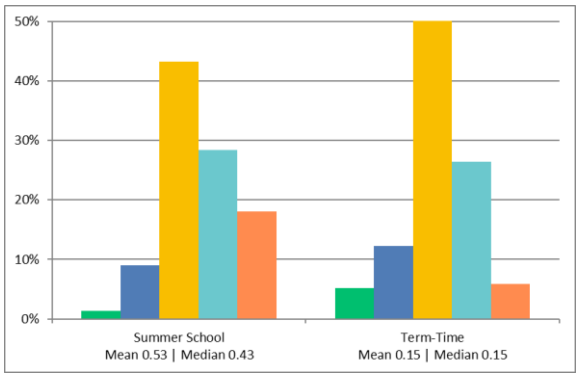

Figure 30 – Social context and interaction outside/external to timetabled classes was beneficial in facilitating my learning.

Value
The *p*-value is 0.9822.

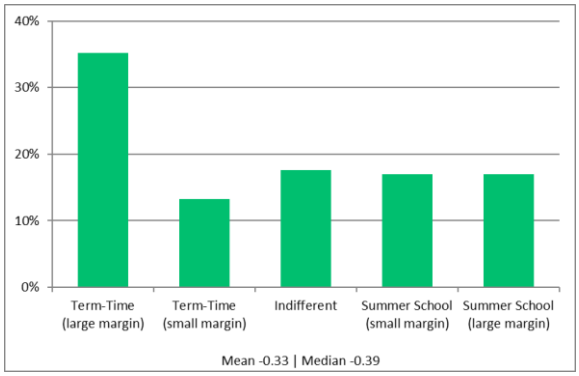

Figure 31 – Which mode of delivery did you find provided you with better value for money, in terms of fees and your own resources?

Qualitative results

Qualitative data arose from open-ended responses. Approximately 1200 comments were extracted, providing over 1900 items, after splitting. Coding produced seven categories, in order of decreasing frequency (appearing in Appendix 2 in Supplementary Material, with descriptors and key words):

Structure, Learning Outcomes, Community, Instructors, Focus, Affordability, Resources.

Structure – Students reflected upon teaching and learning activities, assessments, materials, and resources, and how conducive these were with learning styles, preferences, and capabilities. We identified four sub-categories:

Design, *Pacing*, *Timing* and *Fitment.*

Timing refers to temporal features of delivery, including distribution of classes, deadlines, study periods and breaks. *Pacing* refers to rates at which material is delivered and absorbed. *Fitment* refers to overall balance and alignment, and how well everything fits together to serve aims and outcomes. *Structure* is listed below as a Presage (multi)variable, though the ingredients may be modified when considering experience. For example, an instructor, sensitive to needs and reactions of students, could tailor the pace, slowing down or speeding up (reflecting the fact that the 3P model is an interactive dynamical system with feedback).

Learning Outcomes – We identify two distinct sub-categories:

Quality and *Satisfaction*.

Comments indicated that learning and understanding concepts were often judged against a personal measure of "enjoyability" and "value". The extent to which students felt challenged or motivated was dependent upon the balance of difficulty and quality of the content and assessment, as well as the learning environment. This led to combinations of surface and deep learning strategies. *Learning outcomes* is listed below as a Product (multi)variable.

Community – We identify two distinct sub-categories:

Cohort Numbers and *Interactions*.

There were many comments as to how and why student numbers influenced learning. The overall sense of community was shaped by personal relationships and interactions. *Cohort Numbers* were outside student control and forms a Presage variable, whilst *Interactions* forms a Process variable.

Instructors – Helpfulness and quality of the teaching staff were often paramount. Students mentioned specific teaching staff, styles, characteristics, and anecdotes, having significant impact upon learning. *Instructors* is listed below as a Presage variable.

Focus – We identify two distinct sub-categories:

Motivation and *Task Management*.

Attitudes, in terms of "drive" or "motivation" to study mathematics, were related to ability to focus amidst varying levels of distraction. These depended upon personal circumstances and balance of study, work and life tasks and goals. *Motivation* forms a Presage variable, whilst *Task Management* forms a Process variable, though *Motivation* may be influenced by Process.

Affordability – Students liked to feel that their learning outcomes were commensurate with the effort and financial costs involved; "value for money" becomes an important outcome. *Affordability* is listed as a Presage variable.

Resources – Students commented about the quality, availability, and helpfulness of resources (such as course notes, recordings, lecture slides, exercise sheets, learning management systems and support services), and how effective they were towards supporting and developing their learning. *Resources* is listed as a Presage variable, though may be modified in the light of experience.

The Presage-Process-Product (3P) diagram

Categories and sub-categories were synthesised within a conceptual framework, leading to a 3P model (Figure 32):

- *Instructors*, *Resources* and *Affordability* become Presage variables,
- *Structure* splits into four Presage variables, corresponding to the sub-categories *Design*, *Fitment*, *Pacing* and *Timing*,
- *Focus* splits into two variables, the sub-category *Motivation* becoming a Presage variable and *Task Management* a Process variable,
- *Community* splits into two variables, the sub-category *Cohort Numbers* becoming a Presage variable and *Interactions* a Process variable,
- *Learning Outcomes* splits into two Product variables, corresponding to *Satisfaction* and *Quality*.

There is an underlying *Cultural Context*, within Presage, splitting into two interacting systems:

- *Student Context*, encompassing *Focus* and *Affordability*,
- *Teaching Context*, encompassing *Structure*, *Instructors*, *Resources* and *Community*.

Avoiding congestion, apart from interplay between *Student* and *Teacher Contexts*, arrows between nodes are not drawn. The diagram represents a dynamical system with feedback. One expects, nevertheless, a general flow from Presage to Process to Product. The relative sizes of boxes indicate their prevalence in students' responses.

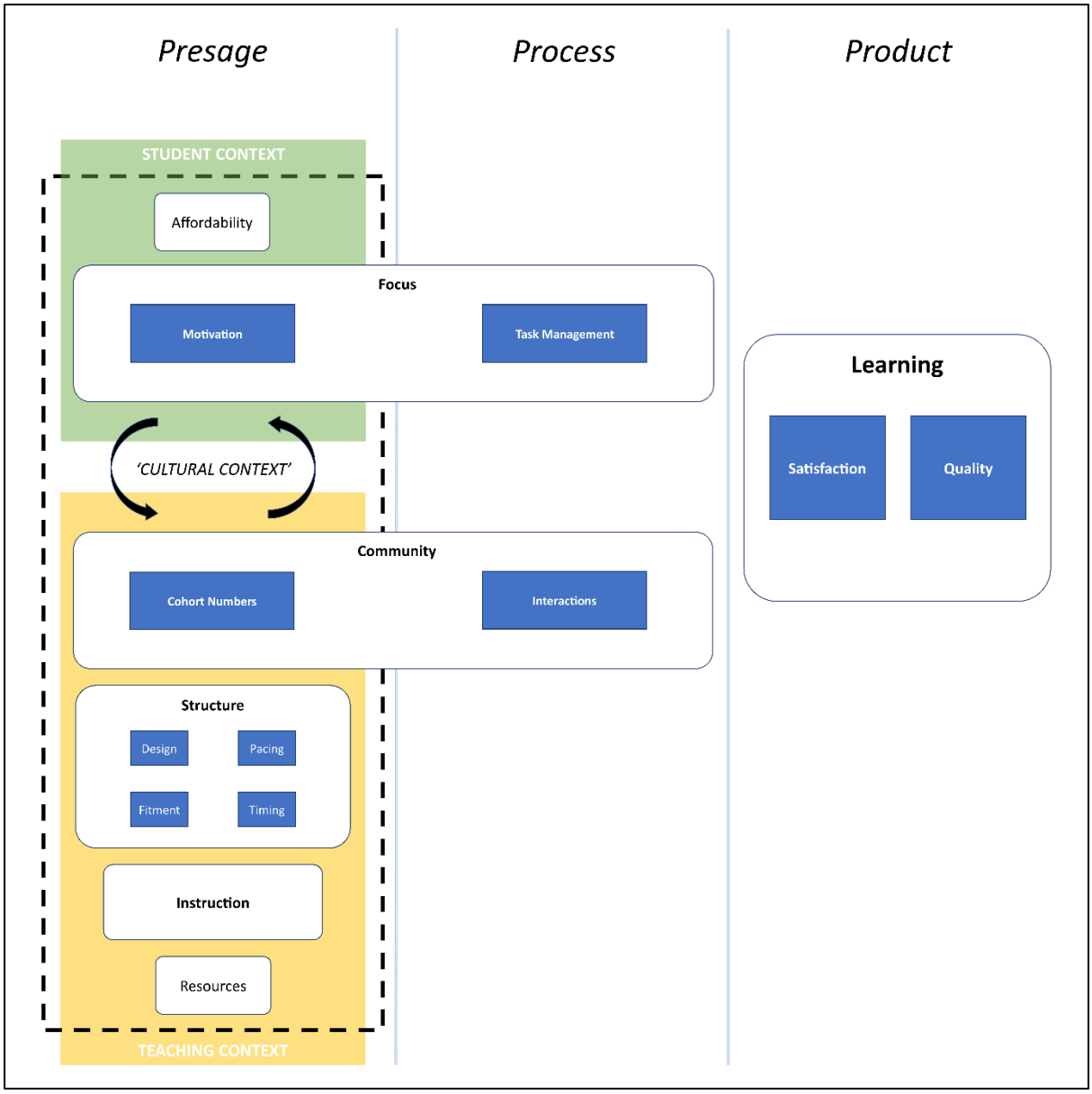

Figure 32 – Culturally modified 3P diagram showing placements and relationships between categories and subcategories.

Discussion

Insights from the quantitative data

There is a clear preference for Summer School (Figures 2-30) except for "value for money" (Figure 31). The preference during semester in terms of value for money should be seen in the context of high fees charged for Summer School over that period.

Enjoyment – Students enjoyed learning mathematics at Summer School:

- about 80% of students preferred Summer School, with 12% preferring term-time (Figure 20),
- over 80% of students agreed that Summer School learning is intellectually rewarding, compared with 40% for term-time (Figure 19).

Instruction – It is striking that

- 75-80% of students at Summer School agreed that feedback was timely, helpful and of high quality, compared with 35-40% for term-time (Figures 3, 7),
- over 85% of students agreed that instructors at Summer School were responsive to students and effective in facilitating their learning, compared with about 50% for term-time (Figures 4, 5, 6).

There were similar percentages indicating satisfaction with the quality of teaching (Figure 2).

Learning – Summer School appeared to provide efficient learning mechanisms:

- over 80% of students indicated that Summer School provided better compatibility with learning styles, compared with about 10% for term-time (Figure 11),
- over 40% of students strongly agreed that they could focus without distraction at Summer School, compared with less than 5% for term-time (Figure 12).

About 55% of students indicated that Summer School provided superior learning outcomes, with less than 10% for term-time (Figure 9). With respect to Summer School,

- over 80% of students agreed that they developed critical skills, were able to focus without distraction and found that learning outcomes were clear (Figures 8, 9, 12).

By contrast, for term-time, about 60% of students agreed that learning outcomes were clear (Figure 7), 45% that they developed critical skills (Figure 8), and 30% that they could focus without distraction (Figure 12).

Classes – Lower enrolments at Summer School created advantages:

- over 90% of students agreed that class sizes were appropriate at Summer School, compared with less than 30% for lectures and about 45% for tutorials in term-time (Figures 14, 15).

Lectures and tutorials blended or morphed well at Summer School, so that their roles tended to become similar, depending on the style and approach of the instructor. That tutorials were successful in both modes is reflected in the fact that

- over 90% of students thought tutorials helped them to learn at Summer School, compared with about 40% in term-time, with over 30% being neutral or indifferent towards tutorials in term-time (Figure 13).

Tutorial sizes at Summer School were capped at 10-15 students, whereas in semester up to 30-35 students could be scheduled for each tutorial. Smaller tutorial sizes improve access to individual attention from instructors. In semester, crowded tutorials may lead to dissatisfaction,

followed by dramatic drop-off in attendance in later weeks. By contrast, a strong relationship or bond between students and instructors appeared to be robust and ongoing for the duration of Summer School.

Assessment – Reactions of students to assessments in both modes were favourable, with similar profiles and about 70-80% agreeing that assessment tasks were challenging and effective (Figures 23-27). However,

- agreement at Summer School was stronger than for term-time, sometimes more than twice as strong.

Written comments varied, reflecting variations in choices made by Summer School instructors, using similar or identical assessment tasks from term-time, or creating their own, or introducing some form of innovation not used in term-time (such as videos or novel group assessment tasks). That strong agreement was higher suggests that there may have been better alignment between learning activities and assessment, reflected in the fact that feedback was more effective in Summer School than in semester (Figures 3, 7).

Motivation – Evidence of a heightened state of motivation during Summer School was prevalent:

- nearly 95% of students agreed (nearly 60% strongly) that they were motivated or driven during Summer School, compared with under 50% (about 15% strongly) during term-time (Figure 16).

Students were able to focus, in a concentrated timeframe, by studying just one or two subjects, allowing them to catch up on a failed unit or accelerate through their degree. Enrolling in Summer School required deliberate effort and organisation at a time when many people were having a break, compared with relatively minimal decision-making when following conventional term-time pathways.

Social – Social cohesion was important:

- over 65% of students agreed (nearly 30% strongly) that social context and interaction was important in facilitating learning *within/during classes* at Summer School, compared with less than 35% (under 10% strongly) in term-time (Figure 29).

At Summer School, students came together with a common purpose, meeting with the same cohort more frequently, in a concentrated timeframe, compared with term-time, where there could be more distractions. The importance of social context *outside/external to classes* was similar between Summer School and term-time, mildly favouring Summer School (Figure 30). This might be surprising, as one would expect more opportunities during term-time for participation in extra-curricular activities. Students at Summer School formed social connections with other participants that persisted beyond formal classroom activities.

Resources – Opinions about availability and effectiveness of resources in Summer School and term-time were similar (Figures 21, 22), though

- strong agreement was higher for Summer School than term-time.

With fewer students, there were certain advantages, such as lecturer access and flexibility, leading to better direction towards resources. The main learning materials supporting units, and Library access, were almost identical for both modes. A deficiency, commented upon, was lack of access to the Mathematics Learning Centre during Summer School, which was, during semester, one of the main fully staffed centres of the University, whose purpose was to provide remediation for students at risk.

Pace and Timing – A common viewpoint is that intensive courses may be too condensed, students needing extra time and space. That was reflected in some of the written comments. However

- 75-80% of students agreed (more than half strongly) that the pace and timing was beneficial for their learning in Summer School, compared with less than 40% in term-time (less than 10% strongly) (Figures 17, 18).

This was one of the most counter-intuitive results of the survey. There is an overwhelming sense that pace and timing of Summer School were well-matched for the demands of units of study and for producing learning outcomes of high quality, confirmed also by the fact that

- about 55% of students thought that units were easier to complete at Summer School than in term-time (over a third by a large margin) (Figure 28).

Insights from the qualitative data

The most intricate part of the 3P diagram (Figure 32) is the arrangement of Presage variables. These form part of the "cultural context", providing a rich and vibrant platform for the student to engage with the material, instructors, and other students, to then move through the Process phase, leading to exemplary or satisfying outcomes in the Product phase. Successful features may vary and can occur in either mode. Students emphasised the importance of *Structure*, *Instructors* and *Resources*. These are mostly stable variables, set by the institution, though facets of *Structure* may be modified or evolve when there are strong communication channels and rapport. Feedback may lead to dynamic changes, so that *Satisfaction* and *Quality* (in the Product phase), or *Task Management* and *Interactions* (in the Process phase), may influence or alter *Pacing*, *Timing* and *Fitment* (in the Presage phase). Comments resonated with observations of Biggs (1996) about how, in CHC (Confucian Heritage Culture) countries, teachers and students interact like fellow-travellers, sharing learning-related beliefs and values:

> *The lecturers and [tutors] are much more responsive to feedback; [they] really genuinely want you to learn and do well and understand the content.*
>
> *... the staff during Summer School were there to show us how to use resources provided as well as going outside what was provided ...*

Students referred to characteristics of their instructors, such as accessibility, approachability, empathy, and helpfulness, which encourage feedback loops, as well as strong personal qualities such as enthusiasm and friendliness. A striking feature was the praise and appreciation that students have for their teachers, mentioning specific experiences that have a long-lasting impact upon their learning and personal development. *Instructors* became a key Presage variable.

Resources emerged as an important variable. Human resources, such as lecturers and tutors, appeared to be more prominent in the minds of students than physical and other resources. This correlated also with the prominence of *Community*, where personal interactions were vital factors in the process of successful and sustained learning. A student wrote:

> *In Summer School everyone was there just to do maths. It felt like a small group on a journey together. Term-time I felt more on my own despite much more people around me.*

Community forms a bridge between Presage and Process, connecting *Cohort Numbers* with *Interactions.* There appears to be a critical mass for developing characteristics of a vibrant and supportive community, including camaraderie, mutual respect, and sense of belonging. Tutorial class sizes were capped at Summer School to 10-15, contrasted by much larger class sizes in semester. Having many people around does not necessarily lead to a sense of community. Loneliness in a crowd is a common phenomenon when individuals are surrounded by strangers and can lead to learning disorders and anxiety.

Learning Outcomes is in the Product phase, where many student responses refer to aspects of *Quality* and *Satisfaction*. Students appeared to be pragmatic, or had to reorganise their lives, to study at Summer School. Favourable outcomes with respect to *Satisfaction* and *Quality* may also influence *Affordability*, in the Presage phase. A student wrote:

> *Summer School is expensive, but it is worth the price because you will get good results and have a better understanding of the subject.*

We have identified Process variables *Task Management*, a sub-category of *Focus*, and *Interactions*, a sub-category of *Community*. *Focus* forms another bridge from Presage to Process. Students embark on their mathematics with certain attitudes and goals, aspects of *Motivation*. This drives them to concentrate on tasks at hand (for example, especially at Summer School, passing a failed mathematics unit of study, or accelerating their degree programme), utilising aspects of *Task Management.* Students often chose one unit at a time at Summer School:

> *This is tied to the fact that during Summer School the unit was my sole focus in study so I was more motivated to understand the content and succeed.*

Students may have to balance many commitments simultaneously:

> *Summer School was all on during a two-day block and in the evening. Term-time at Sydney University almost all maths subjects are spread across the entire week which is horrible for mature age students like myself that have to work.*

The Presage variable *Motivation* has the capacity to evolve through Process (such as successful or appropriate *Task Management*) and within a favourable cultural context. A student wrote:

> *During Summer School I felt motivated to finish everything. During term-time, with many other units at the same time, the assessments were more of a burden than a learning tool … in Summer School I found that I actually [had] the motivation to read them, because I was in a more relaxed state.*

Part of the success of Summer School appeared to be an ability to focus on a limited amount of material, delivered in a relatively short period of time, supported by a community of like-minded learners, interacting with empathic and enthusiastic teaching staff. With correct settings in Presage, there appears to be a smooth pathway through Process towards Product, leading to successful learning outcomes. Intensive modes of teaching during semester have been adopted recently at The University of Sydney, with options for completing units in a half-semester.

One might expect that characteristics related to ability and mathematical background should appear as ingredients of Presage and be surprised that these are missing from our diagram. Such personal characteristics did not feature in comments (though there were many comments about characteristics of the instructors), and therefore do not appear in the categories that arise through coding. There was some degree of homogeneity in the student population that attended Summer School, as there was no opportunity to study Advanced units. However, there was enough variation, between prerequisites for Fundamental and Mainstream units of study, that one might expect levels of preparedness to be an important factor. There seems to be a "blindness" about this issue: students were determined to succeed regardless of background or ability. This resonates with CHC phenomena noted in Watkins and Biggs (1996, 2001b) that there is no impediment to success, given enough effort and a suitable learning environment. This also resonates with findings in Easdown *et al.* (2019) of the tendency of students to perform in higher qualitative phases (SOLO), even when backgrounds suggest an expectation of failure or superficial learning outcomes.

Conclusions

This research expands the scope of previous work, by considering students that took any mathematics units of study offered at The Sydney Summer School in the period 2009-2016. The quantitative data shows, overwhelmingly, a preference for the learning environment provided at Summer School, rather than during semester. These students appeared to

- enjoy learning mathematics and find it intellectually rewarding, finding the quality of teaching and instructors' responsiveness superior at Summer School.
- believe that Summer School was more tailored to their learning styles, with superior opportunities to learn without distraction.
- have high regard for the clarity of learning outcomes and development of critical skills in the Summer School mode.
- appreciate more appropriate class sizes at Summer School, especially where lectures tend to morph into tutorials, contributing to a vibrant learning culture.
- be able to focus, in a concentrated timeframe, on one or two subjects, allowing them to catch up or accelerate through their degree.
- find social interaction within/during classes occurs more prominently in Summer School, providing a sense of community and social cohesion, compared with more distractions in term-time.
- find that, with fewer numbers of students, there is better direction or access to resources.
- with some exceptions, find the pace and timing more beneficial for learning in Summer School.

Comments indicated that features of both modes were important in influencing the quality and satisfaction of learning outcomes. Coding produced seven categories. Relationships between them led to a Presage-Process-Product (3P) diagram, motivated by the culturally modified 3P diagram of Biggs (1996b). This diagram emphasises the importance of presage and temporality, within the context of a vibrant and dynamic community of learners. Successful features of this "culture" may vary from student to student and can occur in either mode. Students referred to their lecturers and tutors in terms of friendliness, enthusiasm, accessibility, approachability, helpfulness, and empathy. Such characteristics enrich the experience and enhance feedback loops in the 3P diagram. Human resources appeared to be more prominent than physical resources; personal relationships and interactions appear to be vital factors in the process of successful and sustained learning.

Part of the success of Summer School appeared to be an ability to focus on a limited amount of material or units, delivered in a relatively short period of time, supported by a community of like-minded learners, interacting with empathetic and enthusiastic teachers. With the right settings in Presage, there appeared to be a smooth pathway through Process towards Product, leading to successful learning outcomes. Personal characteristics relating to ability or levels of mathematical preparedness were absent from comments and do not appear in the categories that arose through coding the data. These students appeared to be determined to succeed, regardless of their background or ability.

Our intention is to develop models that might be useful for educators in course planning or development, or who wish to improve the efficacy or quality of existing courses and programs. The model in this paper highlights the central importance and interaction of key presage variables. It attempts to uncover the dynamics of a culture associated with community of learners, to help make explicit what works and does not work. We hope our model can help orient and smooth the way for novice educators, as well as offer guidance or reassurance for those with more experience. Whilst we believe this study identifies important and possibly universal aspects and dynamics of mathematics education it should be noted that there are certain limitations:

- the cohort did not typically include or specifically address highly talented or gifted students, who might be contemplating Honours or postgraduate research degrees.
- the period 2009-2016 occurred well before the advent and ubiquity of online learning and vast technological advances that have taken place, out of necessity, to deal with the recent pandemic.

Regarding this second point, it would be interesting to test aspects and dynamics of our 3P model, possibly updating or augmenting it, in the context of remote and asynchronous learning. This study is based on the student perspective, and it would be interesting also to perform similar analyses based on perspectives of the instructors, unit of study coordinators, or people involved in remediation or assisting students with disabilities or learning difficulties. Though this study involves mathematics units of study only, in certain learning settings, it would be interesting to test aspects of our model in other disciplines and environments. Our study and model also suggest that there may be a "sweet-spot" in terms of optimising resources and timing, and it would be interesting to see if this phenomenon can be replicated elsewhere.

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