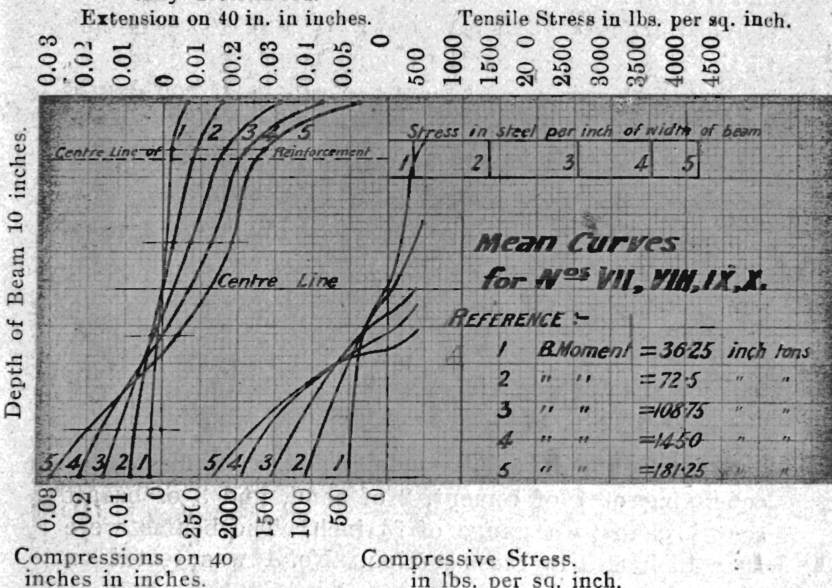
ner; the results obtained in the four beams were very consistent and differed very slightly from each other. Fig. 8 shows the actual lengthening or shortening plotted with reference to the depth of the beam in a similar manner to that employed in the strain curves 1, 2, 3, and 4 in the plain concrete beam Fig. 6. In Fig. 8 five strain curves are plotted for five corresponding bending moments, and the stress curves derived from the strain curves by means of Fig. 5 are complete on the compression side of the neutral axis. The curves on the tension side are necessarily incomplete as Fig. 5 does not furnish the data for continuing the curves beyond the points shown, which is the tensile strength of the plain concrete. The stress in the steel reinforcement is determined from the extensions measured, and the coefficient of elasticity of the steel.

Fig. 8.—Distribution of Strain, and equivalent Stress, over the Cross Section of Reinforced Concrete Beams as experimentally determined.

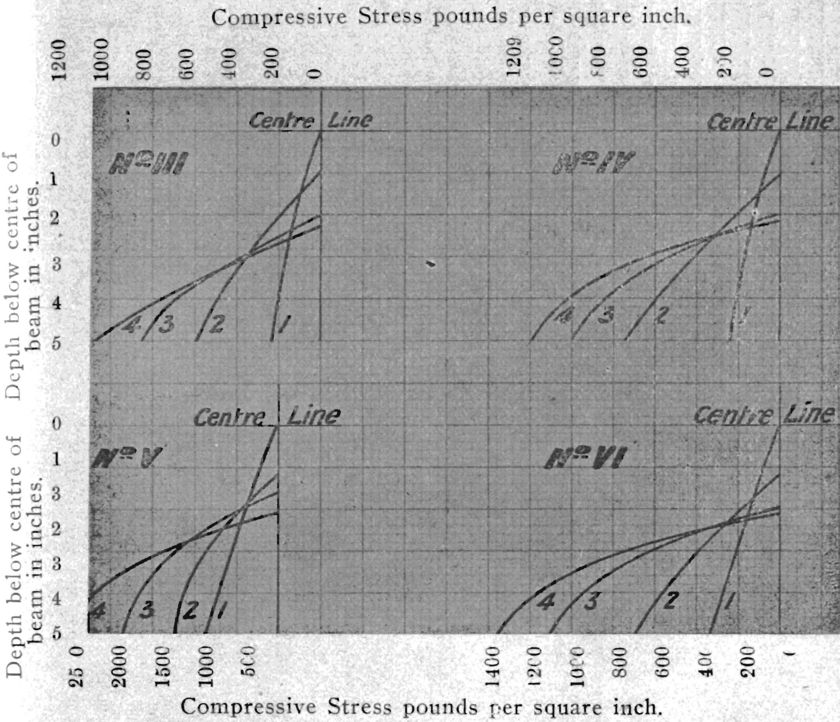


The curves 1, 2, 3, 4 and 5, in the strain diagram on the left of Fig. 8, show that a plane section before flexure is not a plain section after flexure, and that the deviation from the plane is greater as the bending moment increases. Again the neutral axis moves from the centre of the beam towards the compression side as the bending moment increases. The diagram shows that the neutral axis for a bending moment of 181·2 inch tons is 1·9 inches from the centre, and for the mean bending moment obtained, in testing the four beams 199·4 inch tons, the neutral axis would be nearer to the extreme fibre in compression. The stress curves derived from the strain curves are fairly straight for a bending moment about one-third of that producing fracture, but they are curved for greater bending moments, curves 4 and 5 being approximately parabolic. Fig. 9 shows the results of testing beams III., IV., V., and VI., but only the stress curves on the compression side of the neutral axis are shown; they are very similar to the stress curves on the compression side of Fig. 8. The neutral axis moves from the centre of the depth of the beam in all four cases, to more than 2 inches towards the extreme fibre in compression, also the curves 1 are practically straight lines, whereas 2, 3, and 4 are approximately parabolic.

The form of the curve of compressive stress in a reinforced concrete beam tested to the breaking point is therefore fairly represented by a parabolic curve, having its origin in the neutral axis, and its maximum ordinate at the extreme fibre in compression.

Experiments were also made on four concrete beams consisting of 1 of cement, 2 of sand, and 3 of basalt rock broken to a gauge of ¾ inch. The beams were 10 x 10 inches in cross-section. No. 1 was tested on a span of 6 feet centres, and Nos. 2, 3, and 4 on a span of 15 feet centres, the loads were in each case applied in the centre. The deflections were measured, also the extensions of the extreme fibre on the tension side, and the extensions and shortenings of the beam at $1\frac{1}{4}$ inches from the extreme fibre on the tension and compression side of the beam.

Fig. 9.—Curves showing the distribution of the Compressive Stresses over the Cross Section of a Reinforced Concrete Beam as obtained by experiment.



Reference.

No. III. 1 : 2 : 3 :Concrete, three ¾ in. bars.1 Bending Moment = 32.25 in. tons.
No. IV. 1 : 2 : 3 ,, ,, ⅞ in. bars. 2 ,, ,, = 72.5 ,,
No. V. 1 : 2 : 3 ,, ,, ⅞ in. bars. 3 ,, ,, = 108.75 ,,
No. VI. 1 : 2 : 3 ,, ,, 1 in. bars. 4 ,, ,, = 135.0 ,,

Age about 360 days.

Figs. 10 to 13 show the deflections, also the extensions and shortenings obtained in the four beams.

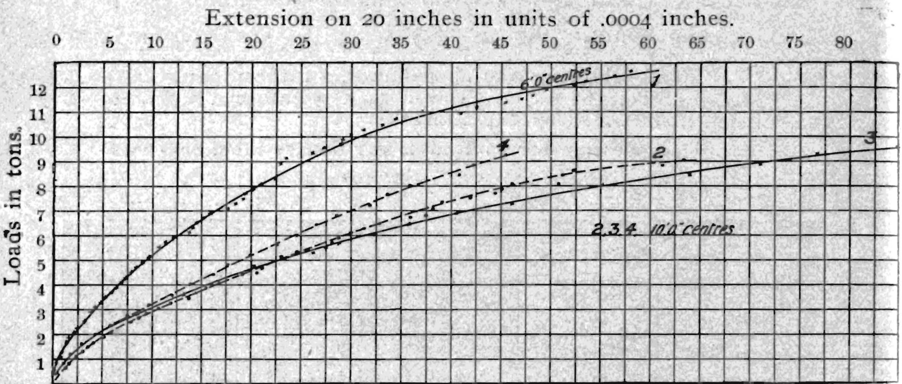


Fig. 10

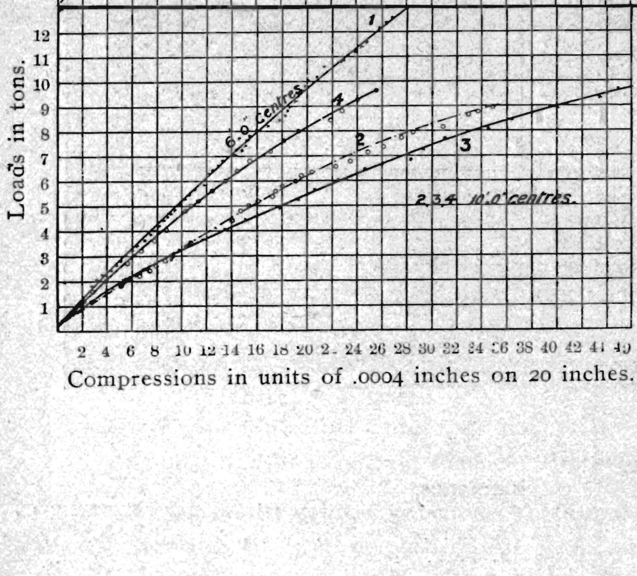


Fig. 11.

Fig. 12.

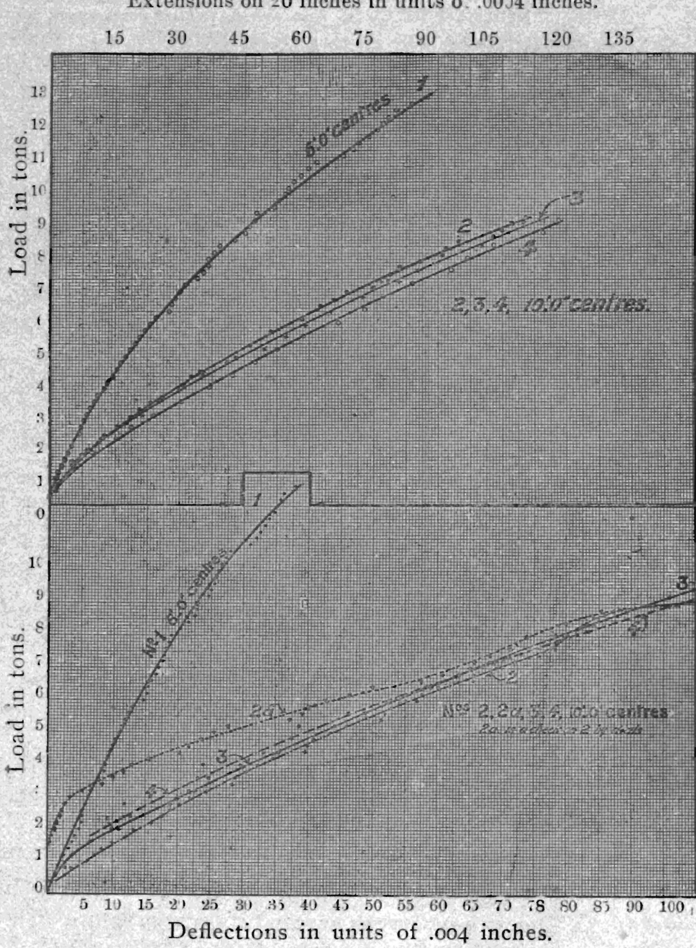


Fig. 13.

It will be observed that the loads and deformations in three beams 2, 3, 4, are very regular, and differ little from each other, and it was considered that the mean obtained would be more useful. The mean of the results on these three beams are given in the following table:

TABLE XIX.—AVERAGE RESULTS OBTAINED FROM TESTING THREE CONCRETE BEAMS, reinforced with three steel rods ⅞ inch diameter, span 120 inches; cross section 10 x 10 inches. Composition, 1 cement, 2 sand, 3 of ¾ inch shivers. Length, 11 feet; weight, 1020 pounds. Age, 105 days.

Load in Tons.	Extension on Extreme Fibre.	Extension on 20 inches 1¼ inch from top of Beam.	Compression on 20 inches 1¼ inch from the bottom of Beam	Deflection in Inches.
·2				
·4	·0005	·00016	·00018	·0038
·8	·0013	·00045	·00079	·0098
1·2	·0022	·00083	·00121	·0169
1·6	·0033	·00114	·00162	·0242
2·0	·005	·00180	·00199	·0401
2·4	·0068	·00252	·00252	·0574
2·8	·0084	·00322	·00309	·0670
3·2	·0102	·00409	·00357	·1011
3·6	·0126	·00508	·00404	·1041
4·0	·0150	·00602	·00457	·1150
4·4	·0174	·00708	·00516	·1432
4·8	·0195	·00779	·00571	·1590
5·2	·0219	·00917	·00621	·1708
5·6	·0240	·01043	·00675	·1952
6·0	·0264	·01141	·00734	·2189
6·4	·0288	·01256	·00793	·2398
6·8	·0312	·01407	·00897	·2662
7·2	·0339	·01550	·00945	·2814
7·6	·0369	·01742	·01016	·3029
8·0	.0400	·01800	·01094	·3135
8·4	·0429	·02191	·01208	·3283
8·8	·0469	·02442	·01318	·3653

NOTE.—The average load producing the first crack was 8·3 tons, and the breaking load 9·6 tons.

The curves showing the extensions, Fig. 10, all show very rapid increase in extension for the gradually increasing loads applied, and corresponding diminution of the coefficient of elasticity of the reinforced concrete in tension. The other stress-strain diagrams show the same peculiarities, but to a lesser extent. In the curves of loads and deflections, Fig. 13, show a very small deflection up to a load of 3 tons, after which the deflections increase more rapidly up to the breaking point. This change in the stress-strain diagram is characteristic, and is seen more or less well defined in all load deflection curves with reinforced concrete beams. The curves, after passing this point, become much straighter, and resemble those obtained in direct tension tests.

Four concrete beams of similar composition and dimensions without reinforcement were tested in a similar manner to the foregoing for the purpose of comparison. Comparing the results obtained from the unreinforced with the reinforced concrete beam the same characteristic differences were observed in the increased loads carried by the reinforced beams, and the enormous increase in deflection before fracture.

The experiments show that the extensions increase in a reinforced beam from the point where the maximum tensile strength of the unreifnorced concrete beam has been attained, to the point where fracture occurs, where it may be ten times as great as in a plain concrete beam. The tensile coefficient of elasticity in a reinforced beam becomes less in proportion to the greater extension, since the tensile strength of the concrete remains constant during the period included between the point where the fracture would occur in a plain beam to the actual fracture in the reinforced beam.

The equations for calculating the position of the neutral axis, and the moment of resistance of a reinforced beam of concrete or mortar may be found in the following manner (Figs. 14, 15, and 16):—

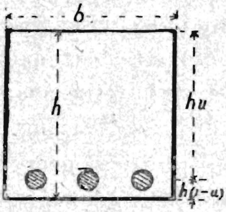


Fig. 14.

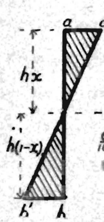


Fig. 15.

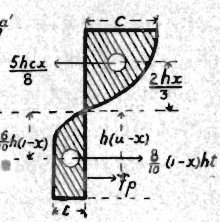


Fig. 16.

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Let h x = the distance from the compression face to the neutral axis of the beam.

h u = the distance from the compression face to the centre of gravity of the reinforcement.

h (1 -- x) = the distance from the neutral axis to the tension face.

h = the total depth of the beam.

Es Ec Et = the coefficient of elasticity of the metal reinforcement, the concrete in compression, and the concrete in tension respectively.

c = the compressive strength in the extreme outer fibre of the concrete.

t = the tensile stress in the extreme outer fibre of the concrete.

f = the stress in the metal reinforcement which should not exceed the elastic limit of the metal.

p = the ratio of the area of the reinforcement to the area of that of the beam, thus, if a = the total area of the metal $p = \frac{a}{bh}$ where b = the breadth of the beam.

We assume that a plain section before flexure remains plain during flexure, or if ab, Fig. 15, represents a line perpendicular to the neutral axis of the beam before flexure, then $a'b'$ represents the position of the line after flexure. This is a usual assumption, but it is not strictly true, as can be proved by measuring the strains on the faces of a beam with delicate extensometers, such as Martens' mirror apparatus; but the assumption is sufficiently approximate in this case, having in view the unavoidable variation in the physical properties of concrete. The form of the stress strain curves obtained by testing beams and prisms in cross-breaking, tension and compression are shown on the numerous diagrams in the paper, from which it will be seen that the area of the curves (Fig. 16) above and below the neutral axis are approximately:—

$$\frac{5}{8} h x c \text{ and } \frac{8}{10} h (1-x) t \text{ respectively.}$$

It is also clear that:—

$$\frac{c}{t}=\frac{E_c}{E_t}\left(\frac{x}{1-x}\right) (1)$$

$$\frac{f}{t}=\frac{E_s}{E}\left(\frac{u-x}{1-x}\right) (2)$$

Equating the tensile and compressive forces:—

$$\frac{5}{8} xc= \frac{8}{10} (1-x) t+pf \ldots (2)$$

Substitute for c and f from (1) and (2) we have:—

For Pread p

$$\frac{5}{8}\frac{E_c x^2 t}{E_t (1-x)}=\frac{8}{10} (1-x)t + pt\frac{E_s(u-x)}{E_t(1-x)}$$

$$\frac{5}{8}\frac{E_c}{E_t} x^2 = \frac{8}{10} (1-x)^2+p\frac{E_s}{E_t} (u-x) \ldots (4)$$

From which quadratic equation x may be found.

Take moments about the point of application of the resultant of the compressive stresses we have the moment of resistance of a unit section:—

$$\frac{M}{bh^2} =\frac{8_t}{10} (1 - x) \left\{ \frac{6}{10}(1-x)+\frac{2}{3} x \right\} +fp\left(\frac{3u-x}{3}\right)$$

$$,, = \frac{8}{150} t (9-8x-x^2) + fp \left(\frac{3u-x}{3}\right)$$

To find the moments of resistance for any intensity of stress in the concrete we must substitute in equation (4) the values of E_s and E_c for the particular stresses c and t and find x which should be substituted in equation (5) to find M. When a crack has developed on the tension face of the concrete t = 0 and equation (4) becomes:—

$$\frac{5}{8}x^2 = p\frac{E_s}{E_c}\left(u - x\right) \ldots (6)$$

Equation (5) becomes:—

$$\frac{M}{bh^2} = fp\left(\frac{3u - x}{3}\right) \ldots$$

Equation (6) may be solved for x thus:—

$$x = \frac{2}{5}\sqrt{\frac{10\,p\,E_s u}{E_c} + \frac{4\,p^2 E_s^2}{E_c^2}} - \frac{4}{5}\,p\,\frac{E_s}{E_c} \ldots (8)$$

If we apply these results to the concrete beams Nos. 2, 3, and 4, recorded in Table XIX and Figs. 10 to 13 we have the following data:—

$$\frac{E_s}{E_c} = 12,\ u = 0{\cdot}9,\ p = 0{\cdot}018$$

Tests made on the steel rods used in the reinforcement give the following results:—

E_s = 31,000,000 lbs. per square inch.

Limit $E = f =$ 42,000 lbs. per square inch.

Ultimate strength = 29 tons per square inch.

Elongation on 10 inches = 25 per cent.

Contraction of area = 69 per cent.

The equations become then:—

$$x = \frac{2}{5}\sqrt{108\,p + 576\,p^2} - 9{\cdot}6\,p$$

$$\frac{M}{bh^2} = \frac{42000}{3}(2{\cdot}7 - x)\,p$$

$$,, = 14000\,(2.7 - x)\,p.$$

$$c = \frac{8\,pf}{5x} = 67200\,\frac{p}{x}$$

$$x = 0{\cdot}41,\ \frac{M}{bh^2} = 576{\cdot}828$$

From which we obtain:—

c = 2950 pounds per square inch.

It is assumed that the extension of the steel rods is the same as the concrete in which it is embedded, and that consequently there is no slip, and that a stress of 42,000 lbs. per square inch was developed in the steel rods at the moment of fracture. If the length of the beam is insufficient to provide the necessary adhesion area to develop this stress, the beam will fail with a smaller load. The average load causing a crack in the three beams was 8.8 tons, and the bending moment consequently 591.360 inch tons.

$$\therefore \frac{M}{bh^2} = 591 \cdot 360$$

The mean extension per inch obtained from the mirror extensometers showed that the elastic limit of the steel was just reached at the moment of fracture. The moment of resistance obtained in the foregoing calculations is sufficiently near the mean result obtained in the testing of the three beams to prove the accuracy of the method adopted in the calculations.

This subject has been investigated in Europe and America by various experimenters. The elaborate work of M. Considère[1] and Professor Hatt[2] may be specially mentioned. The equations obtained by these authorities are very similar to those given by the writer, and are expressed as follows for the sake of comparison:

Considère obtains:—

$$c = \frac{E_c}{E_s} f \left(\frac{x}{u-x} \right) \ldots (1)$$

$$t1\,(-x) + fp = \frac{E_c f}{2E_s} \left(\frac{x^2}{u-x} \right) \Big\} \,(2)$$

$$M = bh^2 \left\{ \frac{t\,(1-x)\,(3+x)}{6} + fp \left(\frac{3u-x}{3} \right) \right\} \,(3)$$

[1] Experimental researches on reinforced concrete by Armand Considère Ingenieur en Chef des Ponts et Chausées, Paris.

[2] See Egnineering News, July 1902.