May 10th, 1917.

THE TRANSMISSION OF POWER BY GEARING.

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The rapid advances which have been made in late years in the application of gearing cannot but fail to be of interest to engineers generally.

With the wide adoption of the electric motor for the driving of machinery of all descriptions, there has been created a demand for improved methods of reducing the high speeds common to motors, and which are necessary if the maximum efficiency is to be obtained, to the more moderate speeds generally required.

The great development of the steam turbine, with similar characteristics as far as speed and efficiency are concerned, has also contributed largely to increased attention being paid to the design and manufacture of gearing.

If it were not for the success with which these investigations have been met, it is doubtful whether the turbine would have met with as large an application to the propelling of high speed passenger vessels, and certainly not to the propulsion of cargo vessels as it now enjoys.

As illustrating the advantages gained by the adoption of reduction gearing for turbines, it may be stated that in the case of the Cunard vessels Mauretania and Lusitania it would have been possible to reduce the shaft horse-power from 70,000 to 57,000 owing solely to the increased efficiency of slower propellors. This is only one portion of the saving which could have been effected, the other being the reduction in the boiler installation owing to the improved steam consumption of the higher speed turbines which might have been fitted. This reduction would be approximately from 60,000 h.p. to 45,000 h.p., and the improved overall efficiency of the complete installation would have resulted in a reduction of about 35 per cent. in the coal consumption. The importance of the position occupied by reduction gearing in marine work will be well appreciated by a study of these figures.

For transmitting the large powers at high pinion speeds which occur in turbine work, the only form of gearing which up to the present has proved successful, has been the double helical, but before dealing with this type it will be interesting to study some of the considerations which enter into the design, and operation of ordinary straight gearing.

The question as to whether the involute or cycloidal form of tooth presented the greatest advantages was the subject of considerable argument in the earlier days when wheels with cast teeth were almost universally used. The principal objection to the involute form was the fact that its use involved greater pressures on the bearings of the mating wheels. On the other hand, the cycloidal shape with its double curve was more difficult to manufacture to even the slight degree of accuracy considered necessary in those days, and it was essential that the pitch lines of mating wheels should exactly coincide if satisfaction was to be obtained. With the involute form this latter condition is not essential, and it was soon realised that the increased bearing pressure was really of small value and importance, and had little effect on the efficiency of the set of gearing.

As a result the involute form is used in the majority of cases nowadays, the cycloidal form being practically only used in cast wheels.

Most probably 95 per cent. of the spur gearing cut in this country is machined to the Brown & Sharpe standards by means of a formed milling cutter.

Some 50 years ago this firm adopted certain proportions for the involute gears manufactured by them, and standardized a compete range of milling cutters for their production. As gearing can be cut on ordinary milling machines provided with a dividing mechanism, and which machine can also be used for a multitude of other purposes, it can be readily understood that the Brown & Sharpe standard form of involute tooth has been largely adopted. The pressure angle chosen was 14½ deg., the length of the teeth above the pitch line or the addendum being .3183 of the pitch, and the length below or dedendum .3683 of the pitch, with a total length of tooth of .6866 of the pitch.

In order, however, to make all gears interchangeable between the limits of a twelve-tooth pinion and a straight sided rack, it was necessary to modify the involute curve to avoid the interference which occurred between the points of the rack and the flank of a twelve-tooth pinion with this pressure angle and dimensions of teeth. This modification also extended to a reduction of the undercutting occurring in a pinion with a small number of teeth.

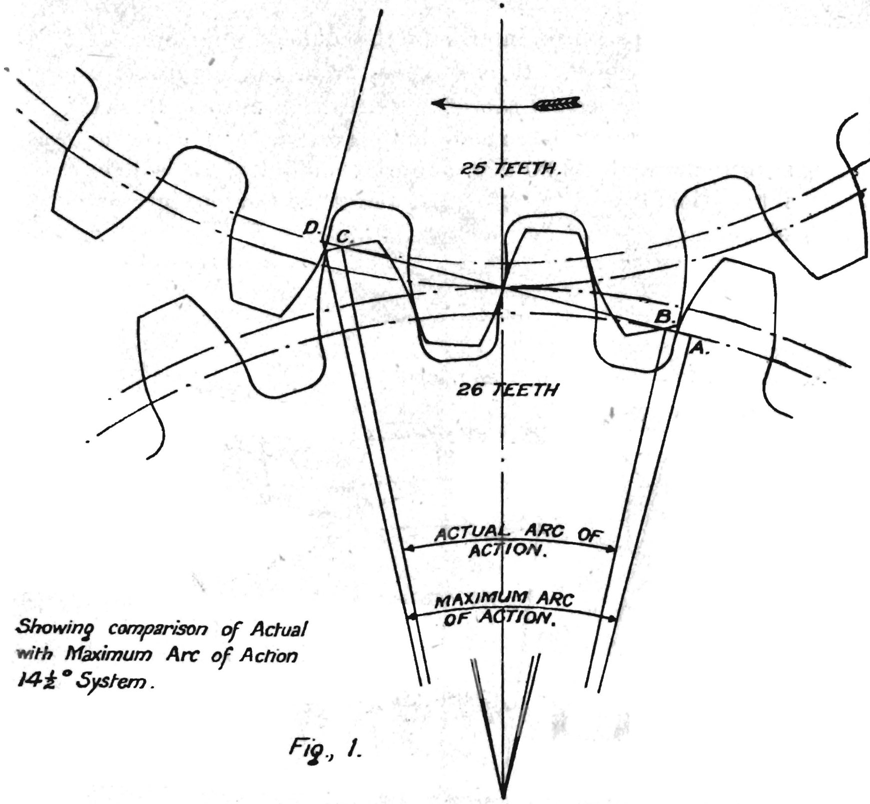
Consideration of these defects of the involute teeth of 14½ deg. pressure angle led to the adoption of greater angles and shorter teeth, and to an increasing extent the pressure angle of 20 deg. is being adopted, while some recognised experts recommended even the greater angle of 22½ deg.

The shorter teeth are generally known as stub teeth, the proportions generally chosen being addendum .25 pitch, dedendum .3 to .32 pitch.

As increasing accuracy of workmanship was demanded gear-cutting machines which generated the involute curve as the teeth were cut were produced, and their use led to a more general adoption of the larger pressure angle, since the curves produced were true and unmodified involutes. With a 20 deg. pressure angle a 14-tooth pinion will gear with a straight sided rack without interference. Merely shortening the teeth will not avoid this. If a 12-tooth pinion is desired as a minimum, an angle of 22½ deg. must be adopted.

The leading advocates of the stub tooth system have been the Fellowes Gear Shaper Co. in America, and Mr. Joseph Adamson in England, and in this connection I would refer members to the excellent paper read before the Institute of Mechanical Engineers in May, 1916, by Mr. Daniel Adamson. To such an extent has the stub tooth system progressed in America that it is stated that at a recent American Motor Car Show nearly 85 per cent. of the cars shown were provided with gears of these proportions. In addition to the fact that stronger teeth are obtained, making it possible to use a greater number of teeth in a given wheel, increasing their durability and smoothness of working, a careful study of the relative movements of the teeth in contact shows that there is an increase of rolling contact between teeth and a reduction in the sliding action which must always take place between gear teeth in mesh.

The following diagrams taken from a publication by the Fellowes Gear Shaper Co., and which were also given in a previous paper by the author, read before the Electrical Association, satisfactorily demonstrate the improvement in action and the reduction of sliding motion, and therefore wear, consequent on the adoption of the greater angle.



Showing comparison of Actual with Maximum Arc of Action 14½° System.

Fig., 1.

Figures 1 and 2 show comparisons of the tooth form of a set of gears of the standard and stub tooth form respectively, the driver having 25 and the driven 26 teeth. If, in the diagrams, the gears are supposed to rotate in the direction of the arrows their point of contact always lies somewhere along what is known as the "line of pressure," this being indicated by the line A.D. in both diagrams. Since this line of pressure is drawn at right angles to the tangent of the pressure angle of the teeth, it will be seen that the pressure angle determines the length of the "line of pressure," and this is apparent from the diagrams.

The point of contact begins on the line of pressure at the intersection of the outside diameter of the driven wheel with the line of action, and ends at the point where the line of pressure intersects the outside diameter of the driving wheel. It is obvious from the diagrams 1 and 2 that the actual arc of action in the normal Brown and Sharpe tooth is considerably greater than with the stub tooth with its 20 deg. angle of obliquity. It will be interesting to see the effect of the reduction in this arc of action.

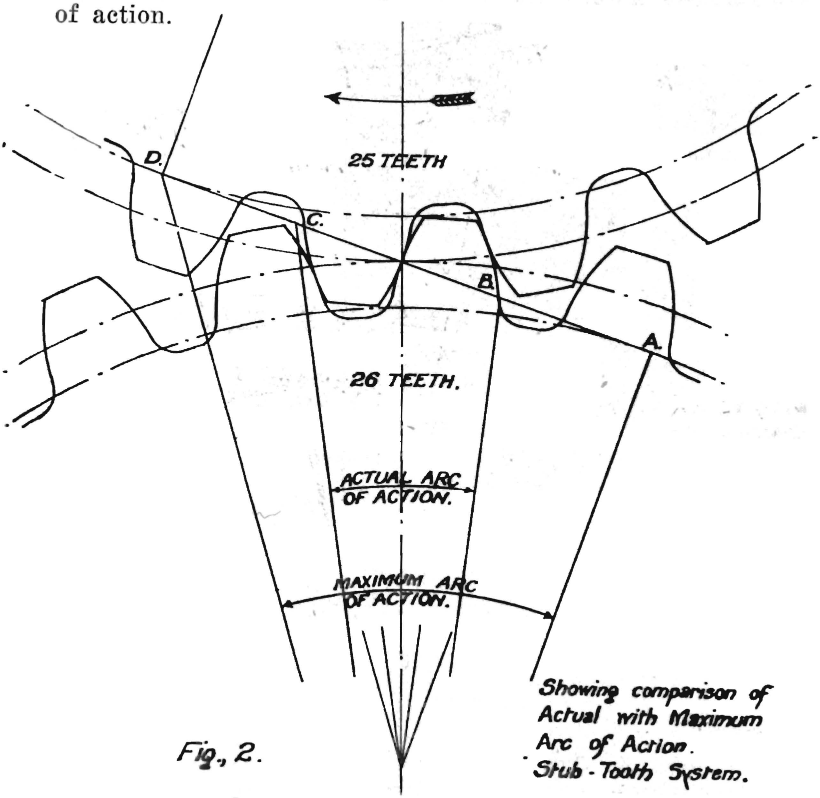
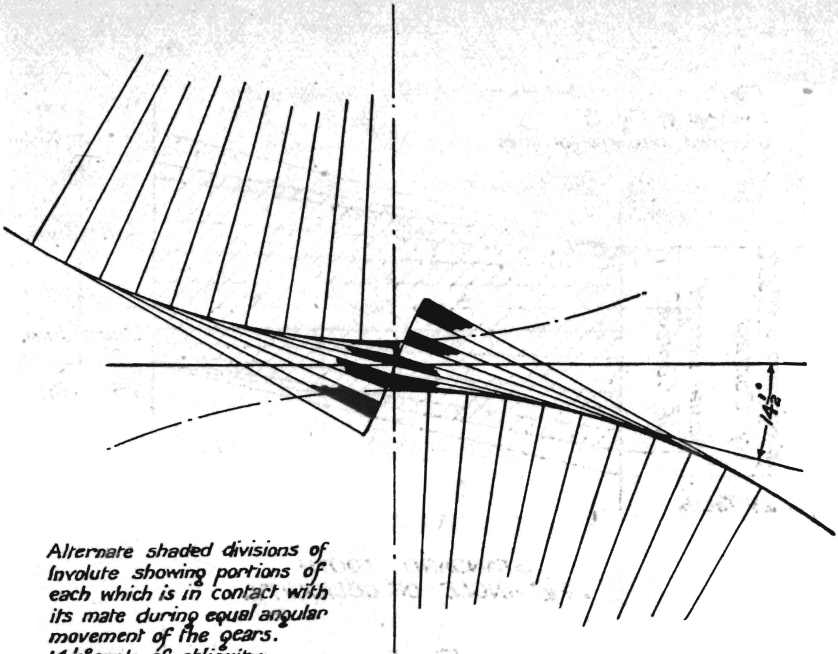


Fig., 2.

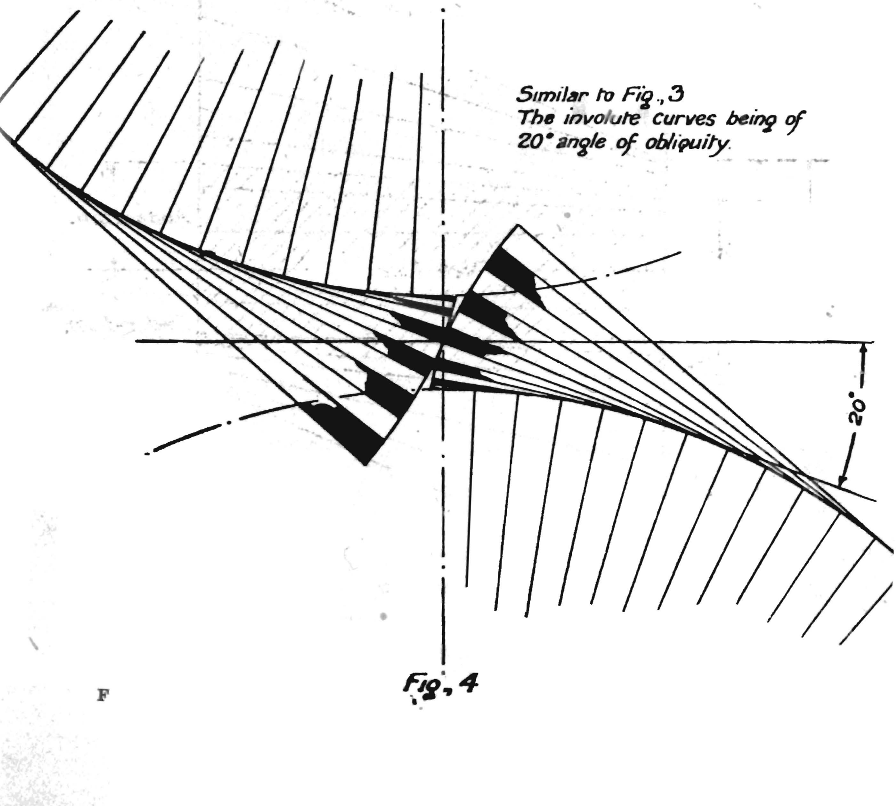
Showing comparison of Actual with Maximum Arc of Action. Stub-Tooth System.

Figure 3 shows an involute curve of 14½ deg. angle of obliquity of each of the gears, and Figure 4 similar curves for a 20 deg. angle, the curves being drawn of a length equal to the maximum arc of contact, and to the same scale as Figures 1 and 2. The alternate shaded divisions of the curves show the portion of each tooth that is in contact with its mate during an equal angular movement of the gears.



Alternate shaded divisions of Involute showing portions of each which is in contact with its mate during equal angular movement of the gears. 14½° angle of obliquity.

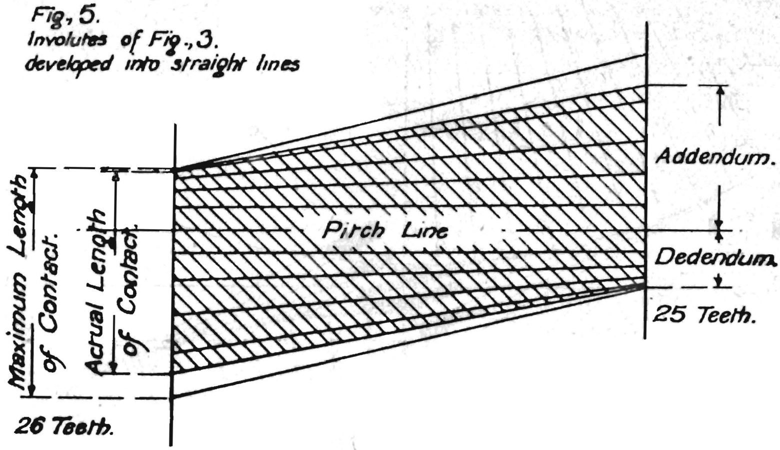
Fig., 3.



Similar to Fig., 3
The involute curves being of 20° angle of obliquity.

Fig., 4

Fig. 5.
Involutes of Fig. 3.
developed into straight lines



STANDARD TOOTH.
$14\frac{1}{2}°$ ANGLE OF OBLIQUITY.

Fig. 5.

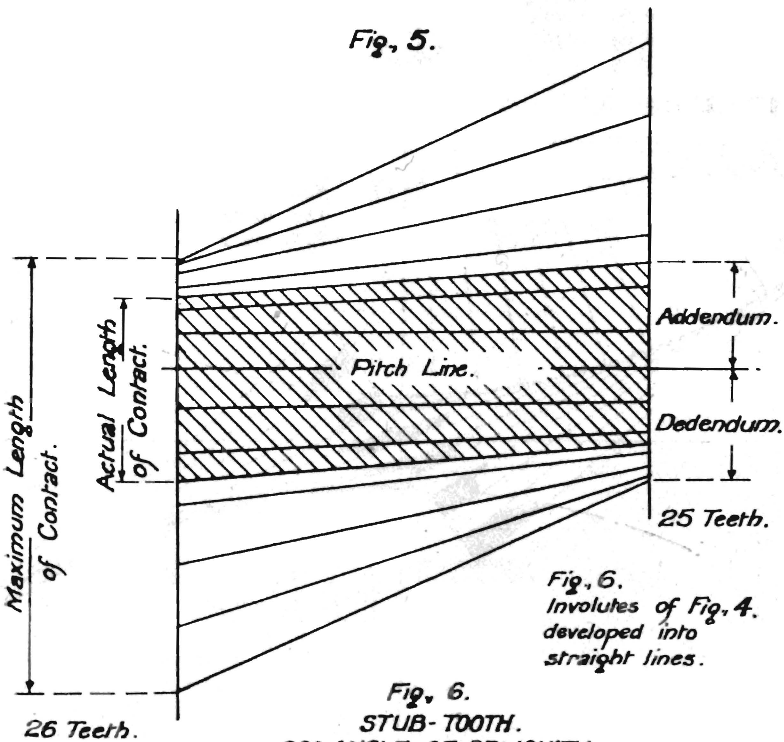


Fig. 6.
Involutes of Fig. 4.
developed into
straight lines.

Fig. 6.
STUB-TOOTH.
20° ANGLE OF OBLIQUITY.

Figures 5 and 6 show the same involute curves developed into straight lines, the points corresponding to the divisions of figures, 3 and 4 being connected by straight lines. It will be noted that, although the divisions of the base circles are equal, those of the involute decrease as the base circle is approached, showing that wear is concentrated at the point.

When, moreover, it is considered that contact takes place between the point of one tooth and the flank of its mate, it will be readily seen that the conditions are not ideal. By a comparison of Figures 5 and 6 it will be seen that the portions in actual contact, denoted by the shaded lines, includes in the case of the 14½ deg. tooth cross lines that have considerable angularity, showing excessive sliding action, while the corresponding lines of 20 deg. teeth are more nearly parallel, denoting that the action is more nearly a rolling one. Due to the reduction in the actual arc of contact with the 20 deg. angle and the removal of the point of the tooth, which wears out the flank of its mate, the adoption of the stub tooth reduces sliding friction and increases the efficiency of the gearing.

At the same time the curves just referred to will be of interest to those who have been under the impression that with involute curves in a set of gears the action is entirely a rolling one.

They also serve to illustrate the fact that a set of gears which does not run well when first installed, due to the curves being improperly shaped, will never improve in service, and that the involute curve in straight teeth is gradually destroyed by wear.

The diagrams, Figures 1 and 2, show the comparative proportions of the standard 14½ deg. tooth and the 20 deg. stub tooth, and clearly illustrate the differences in form.

While considerable improvement in wearing characteristics can be obtained by this departure from original practice, noise in operation can only be reduced by accuracy of workmanship.

Comparatively quiet running cannot be obtained from straight metal gears cut on an ordinary milling machine except at the most moderate pitch line speeds. Even below 600 feet per minute the noise becomes noticeable, and at 1200 feet is most objectionable.

This noise is due to the heavy pressures set up in the teeth due to their inaccuracy, and the author would refer members to the well-known investigation of this subject by Lasche, and detailed in article in the Zeitung Des. Ver. D.I. of 1899.

He therein shows that in a given pair of wheels transmitting 75 h.p., and with an error in pitch of .02 in $1\frac{1}{2}$in., or 1.3 per cent. of the pitch running at 2460ft. per minute, the loading on the teeth was 28 times the normal tooth pressure. He further shows that excess loads due to errors in pitch vary as the square of the pitch line velocity.

The extent of this increase of load is of course due to the continual acceleration and de-acceleration of the wheels due to the uneven angular velocity caused by the errors in pitch.

Mr. Daniel Adamson in his paper, previously referred to, states that he found the average error between adjacent teeth in a cast wheel of $1\frac{3}{4}$in. pitch to be in the neighbourhood of .4 per cent. of the pitch in an ordinary cast wheel, in a machine-cut wheel .2 per cent. of the pitch, and in a generated wheel only .01 per cent., or half of the error in the wheel with the milled teeth. These figures refer to the average error only, the corresponding maximum errors for cast teeth being 5 per cent., ordinary machine-cut wheels 1 per cent., and generated wheels .08 per cent. These figures show that for high speed work the milled tooth occupies a very inferior position to the generated wheel, and cannot be expected to run as quietly.

The noisy operation of inaccurately-cut gearing can be reduced by the use of rawhide, paper, or linen pinions, but while these, because of their nature, do not emit as much noise, they do not reduce vibration to any great extent, and have other limitations.

The principal of these in the rawhide gear is that ordinary hydro-carbon oil is most detrimental in its effect, and its use will be followed by a rapid destruction of the pinion. Paper and linen pinions do not suffer from this defect, but blanks of these two materials seem to be generally unobtainable in this country. A solution of the problem of obtaining silent operation would appear to be by introducing a certain amount of flexibility into the teeth of the wheels in gear or in their bearings. This has been done with striking results in the "Laminated" gear. The Laminated gear is made up of a number of thin saw steel plates in which teeth are cut. The plates are then assembled with every alternate plate, half a tooth pitch ahead of its neighbour. In order to give the necessary side clearance between teeth, thin distance pieces of a softer metal are inserted between plates, the whole being mounted on a cast-iron spider with side clamping plates, and rivetted together. The slide exhibited shows an example of this type of gearing, and in several small drives of which the writetr has had experience the results have been most satisfactory, the teeth showing practically no wear after several years of almost continual service, together with silent operation.

In addition to the advantages gained by the flexibility of the teeth, the silence is also attributable to the fact that there are a greater number of teeth in contact than in an ordinary gear wheel of equal pitch, and their action more nearly approaches that of helical wheels, but with contact surfaces at right angles to the plane of the wheels.

Non-staggered pinions are also made up to mate with ordinary spur gearing, and to take the place of rawhide pinions, and if the quietness of operation claimed by the makers is obtained in practice, the fact that flexibility of teeth reduces noise by permitting a more equal distribution of stresses may be considered proven. The fact that grease is retained in the space between the teeth no doubt contributes to their silent operation.

The formula most frequently used in the design of ordinary straight-out spur gearing is probably that due to Lewis, viz.:—

$$P = y \times p \times b \times f$$

Where P = pressure on teeth in pounds.
p = circular pitch in inches.
b = breadth of face.
f = safe stress in material.
y = a constant dependent on number of teeth and form of same.

This formula is derived from the simple one of a cantilever of span h, breadth b, depth d, loaded at the end with a force P. If f is the stress produced at the weakest point of the tooth then

$$Ph = \frac{b\, d^2\, f}{6}$$

For any given material, form of tooth and number of teeth, the relationship between the depth or thickness of the tooth and its height or length to the pitch is a constant, and it was from a series of drawings of teeth of the involute, cycloidal and radial flank systems that Lewis derived his constant y, tables of which will be found in most pocket books.