POST-SECONDARY STUDENTS' LEARNING OF DESCRIPTIVE STATISTICS THROUGH STORY-BASED TASKS

Collette Lemieux[a], Olive Chapman[b]

Presenting Author: Collette Lemieux (clemieux@mtroyal.ca)
[a]Bissett School of Business, Mount Royal University, Calgary, Alberta T3E7P7, Canada
[b]Werklund School of Education, University of Calgary, Calgary, Alberta T2N1N4, Canada

KEYWORDS: Descriptive statistics, Understanding, Story-based tasks, Post-secondary students

ABSTRACT

This paper reports on one aspect of a larger project that investigated the use of inquiry-oriented, story-based tasks in teaching statistics. Specifically, its focus is on identifying levels of understanding of descriptive statistics concepts that students in a first-year university business statistics course were able to develop through their engagement in a story-based task. Understanding was framed by Skemp's (1976) theoretical perspectives of instrumental and relational understanding. Data sources consisted of students' written responses to the story-based task during the course. Findings indicated that most of the students were able to develop instrumental understanding and a partial level of relational understanding of the concepts. In general, findings suggest that learning statistics through stories has the potential to have a positive impact on students' understanding of concepts in areas that previous research suggests is difficult for students to learn.

Proceedings of the Australian Conference on Science and Mathematics Education, 30 September - 2 October 2020, pages 116-122, ISBN Number 978-0-9871834-9-1.

INTRODUCTION

Given the importance of statistics in today's data driven world, more than a decade ago, the first Guidelines for Assessment and Instruction in Statistics Education (GAISE) report was published to provide guidelines on best practices in statistics education (Garfield, Aliaga, Cobb, Cuff, Gould, Lock, Moore, Rossman, Stephenson, Utts, Velleman, & Witmer, 2005). These guidelines are part of a reform movement in statistics education and suggest that instructors should use active learning, real data sets, and technology to aid students in developing statistical knowledge, reasoning, and thinking. But there have been difficulties in successfully converting these guidelines to practice (Tishkovskaya & Lancaster, 2012), with many innovations in statistics education being based on intuition rather than on research (Ramirez, Schau, & Emmioglu, 2012). Thus, although there have been changes to how statistics has been taught in recent years, there is still a need for research on how to effectively support students' understanding of statistics concepts using suggestions of best practices from the reform movement. This paper reports on one aspect of a larger study that contributes to this need through an investigation of the use of inquiry-oriented, story-based tasks in teaching statistics. The aim of this aspect of the study was to determine the levels of understanding of descriptive statistics concepts that students in a first-year university business statistics course were able to develop through their engagement in a story-based task, which provide an authentic context for them to actively explore and learn the concepts.

NATURE OF STORIES

In this study, stories are defined as narratives that have a clear beginning and end, and tell a sequence of events with a character that is driving the events towards a solution to a problem or conflict (Egan, 1986). They have characters, plots, context, conflict, imagery, emotions, and humour (Roberts & Stylianides, 2013; Zazkis & Liljedahl, 2009). This interpretation of story guided the development of the story-based tasks for the intervention. Storytelling could allow students to connect a statistical skill to "a particular human hope, intention, fear, or whatever, then [they] can embed the skill in a context that is meaningful" (Egan, 1986, p. 77). Thus, stories provide the opportunity to ground the abstract statistical concepts in a meaningful context, which connects the concepts to students' lives and future professions. For this study, defining the tasks as stories resulted in detailed description of the context and problem of the story; development of characters with distinct personalities; addition of dialogue between the characters; use of informal language that fit with the

personalities of the characters; physical descriptions of settings and characters; and sequential progression of events over a period of time.

THEORETICAL PERSPECTIVES OF UNDERSTANDING

The aim of this study was to determine students' levels of understanding of descriptive statistics. In order to do this, we framed the work theoretically by Skemp's (1976) notion of instrumental and relational understanding. Skemp defined instrumental understanding as the ability to perform an algorithmic procedure to solve a problem, but without knowing the reasons for the steps or "rules without reasons" (p. 20). In the context of statistics, students with instrumental understanding would know how to do a calculation or follow a procedure, but they would have difficulty interpreting the results. Further, they may know the definition of a term, but could not apply the term in practice or know why the definition was appropriate. Relational understanding, on the other hand, is "knowing both what to do and why" (p. 20). In the context of statistics, students with relational understanding would know what statistical procedure to use and why it is appropriate to use in multiple contexts; how to perform the procedure in multiple contexts; how the procedure works; how to interpret the results; the limitations of the procedure; and how the various procedures are interconnected. They would also know the reasoning behind the concepts. Both instrumental and relational understanding are relevant to this study because students can demonstrate either or both to various levels in their learning of the statistics concepts through the story intervention.

LITERATURE REVIEW ON STORY-BASED TASKS

Though stories are used to teach mathematics in elementary school (e.g. Roberts & Stylianides, 2013), they are rare post-secondary education. We found only two studies that utilised story-based tasks to explore statistics at the post-secondary level. Blackburn (2016) created a story about a fictionalised fish farm to contextualise statistical concepts in a business setting. The stories have characters such as "Freaky Fish" who explained statistical concepts (p. 3). He found that students reported that they were engaged in the learning through the intervention and believed that they would retain the material longer than if traditionally taught. But students did not perform better on tests after learning through stories. D'Andrea and Waters' (2002) study examined how the use of short stories impacted students' statistical anxiety and attitudes about the usefulness of statistics. The authors wrote a series of short stories that involved solving crimes. The researchers found that, though levels of anxiety decreased, students still had difficulty believing in the relevance of statistics to their everyday lives. Based on this review, further research into post-secondary students' learning through stories is appropriate.

LITERATURE REVIEW ON STUDENTS' LEARNING OF DESCRIPTIVE STATISTICS

Here we highlight studies on three key topics of descriptive statistics that are relevant to our study: visual representations, measures of central tendency, and measures of variation. These studies tend to highlight post-secondary students' difficulties or misconceptions in learning these topics. They indicate that students have difficulty with creating and understanding visual representations of data, such as histograms. For example, Zaidan, Ismail, Yusof, and Kashefi (2012) found that when students were asked to label histograms of the salary of individuals over the age of 40, students labelled the horizontal axis as salaries and the vertical axis as age. Students also have issues reading histograms. For example, Kaplan, Gabrosek, Curtiss, and Malone (2014) found that students identified the mode of two data sets being the same based on the heights of the bar being the same rather than where the bars were positioned.

While students have some understanding of measures of central tendency such as the mean, they have conceptual difficulties with it (Dubreil-Frémont, Chevallier-Gaté, & Zendrera, 2014). As Mathews and Clark (2007) found, for students who received an A in a university statistics course, some could give a definition of the mean, while others confused mean with the mode and proportion. Zaidan et al. (2012) also found that post-graduate students' difficulties included believing that if the mean was three, then the data set had to be either composed entirely of threes (e.g., 3, 3, 3) or multiples of three (e.g., 3, 3, 3 or 9, 9, 9).

Unlike measures of central tendency, most post-secondary students studied struggled with even a superficial understanding of measures of variation such as standard deviation. Gougis, Stomberg, O'Hare, O'Reilly, Bader, Meixner, and Carey (2016) found that only 17% of science students in their study understood variation, while Mathews and Clark (2007) found that none of the undergraduate students in their study who got an A in a statistics course had even a partial surface understanding of

standard deviation. In particular, they found that students could calculate the standard deviation, but did not know what the calculated value meant and could not provide a definition for the standard deviation. Common issues in understanding standard deviation include that it is found by determining the distance between the data values in the sample (Mathews & Clark, 2007); it is the distance between the mean and one data value in the sample (Lavy & Mashiach-Eizenberg, 2009; Mathews & Clark, 2007); and the standard deviation and the mean are equal (Chan & Ismail, 2013).

While most studies on student learning of statistics, as in the above review, tend to focus on students' misconceptions or difficulties, this study explored the understanding the students were able to construct with less focus on the nature of their difficulties or misconceptions. Thus the aim of this study was to highlight what they were able to do through the story-based tasks.

RESEARCH METHODS

A qualitative case study methodology (Stake, 2005) was used for this study to investigate students' understanding of descriptive statistics based on their participation in one course in one semester.

THE COURSE AND STORY-BASED TASKS

The course is a multi-section algebra-based statistics course that is intended for first-year business students at a Canadian university. It is scheduled for four hours per week during a thirteen-week semester, with two classes per week. Sampling techniques and descriptive statistics, one unit of the course, serves as an introduction to the course. Topics included visual representations, measures of centre, measures of variation, and measures of location. One section of this course, which comprised 40 students, was used for this study. The instructor was a tenured faculty member with over fifteen years of experience teaching statistics. He worked with the first author (researcher) in piloting an initial version of the story-based tasks, which provided him with adequate experience teaching with such tasks for the study.

The story-based tasks were developed to engage students in meeting the learning goals of the reform movement in statistics education by encouraging the development of statistical knowledge, reasoning and thinking through active learning, effective use of technology and real-world contexts. They were written as short stories and were around 10 to 12 pages long. The stories were fictional, but set in realistic situations. Each story had a problem to be resolved through some form of statistical analysis. The stories were intentionally left incomplete with 12 to 14 prompts per story that required students to write dialogue between the characters. These dialogues involved interpreting statistical measures, drawing conclusions from the statistical analysis, explaining reasoning for why specific statistical measures were chosen, and explaining aspects of the statistical concepts covered in the stories (see Lemieux, 2020 for an example of a story-based task). As such, the stories were written both as a teaching tool (teacher as storyteller) and as a cognitive tool (student as storyteller/narrator; Roberts & Stylianides, 2013).

Each story focused on one major unit of the course and had a unique context in which the students explored the statistics topic through the story. In this paper, we focus only on the story-based task called *Bob's Bikes*, which was used at the end of the unit on sampling techniques and descriptive statistics. This task provided students with the opportunity to consider how the separate concepts could be used together to address one problem. These concepts included: visual descriptive statistics including histograms and box plots; measures of centre including mean, median and mode; measures of variation including range, standard deviation, interquartile range, and coefficient of variation; and outliers. The story of *Bob's Bikes* is that of three accounting articling students, Jolene, Franca and Bart, who are asked to determine whether an inventory system should be repaired immediately or if the cost could be deferred to a later date. The first part of the story has the characters describing sampling techniques, which they then implement. Once they have their sample, students found various descriptive statistics that they interpreted and then considered as a whole to determine whether the system should be repaired or not. In responding to the story, the students explored sampling methods, finding and interpreting descriptive statistics, and arriving at a conclusion based on the sample.

PARTICIPANTS

The participants were 19 of the post-secondary students enrolled in the statistics course, who volunteered to be involved in the study. The majority of students were in the first-year of their

business programs with a few being in their second- and third-year. This is the only post-secondary mathematics course required for the business degree and the only pre-requisite for it is grade 12 mathematics. Thus, the majority of students did not have prior experience with the majority of the content covered in the course.

DATA COLLECTION

Data sources for the larger project included the participants' written responses to each story-based task and a corresponding follow-up task, which were collected shortly after the due date for each task in the course. Participants worked in groups of two on these tasks. For the story-based tasks, their responses were in the form of dialogues embedded within the stories that demonstrated their understanding of the statistics concepts associated with the stories. The follow-up task for each story-based task was assigned to students immediately after they completed the story-based task. Students completed the tasks outside of class time and worked in the same groups as the story-based task. These tasks required that the students apply their understanding of the statistical topics covered in the story-based task in a new and different context, which provided another way of demonstrating their understanding. For example, in the story-based task students explained variation within the data for the context of inventory errors, but in the follow-up task they explained it for the context of out of pocket dentist costs. Responses to 14 questions per pair (seven from each task) were analysed. The following is an example of students' dialogue, within the context of the story *Bob's Bikes*, explaining what the absence of outliers means within the context of the inventory problem.

> *Bart:* I thought that there would always be outliers in box plots?
> *Jolene:* No, like Franca said, outliers only exist when there are extremes or when something doesn't fit in with the rest of the data. This just means that Bob's inventory system wasn't undervaluing the products with huge differences, that the amount the products were being undervalued by was consistent. That's why we have no outliers, everything fits in and nothing extremely unusual is happening with the inventory. Sure it is undervaluing the inventory, but not by an insane amount, so that is why there are none present in this data.

For the part of the study reported here, the focus was on the participants' dialogues for the *Bob's Bikes* story-based task and response to the related follow-up task.

DATA ANALYSIS

Data analysis for the larger study consisted of three stages to identify the level of students' understanding of the statistics concepts demonstrated in the data. The first stage involved open-coding of this data for common ways participants demonstrated understanding (if any) for a concept to get an initial idea of the participants' understanding. For example, we identified the different ways that participants explained how they found the outliers. An example of a common way was looking at the box plot. The second stage of analysis involved coding the common ways participants responded in the tasks based on our disciplinary knowledge and Skemp's (1976) framework for relational and instrumental understanding. Finally, the third stage of analysis involved looking across the story-based and follow-up tasks to determine if any common levels of understanding emerged from the data. The focus was on categorising the information from the codes found in stages 1 and 2 by looking for themes and patterns related to different levels of understanding. For example, a common way that participants demonstrated instrumental understanding was by defining a term in their own words. Coding and themes were done by one researcher and checked by the other. Discrepancies were discussed and resolved based on supporting or lack of supporting evidence. The findings presented in this paper focus on the participants' collective levels of understanding of the statistics concepts related to descriptive statistics.

FINDINGS: UNDERSTANDING OF DESCRIPTIVE STATISTICS CONCEPTS

The findings consist of the levels of understanding, specifically, instrumental and relational understanding, demonstrated by the participants for each of the following concepts for descriptive statistics: visual descriptive statistics (histograms and box plots), measures of centre (mean, median and mode), measures of variation (range, standard deviation, interquartile range, coefficient of variation), outliers, and conclusions made based on descriptive statistics. In presenting the following overview of the key findings, we use *most* for over 75%, *many* for 50-74%, and *few* for less than 25% of participants.

All participants demonstrated instrumental understanding in following algorithms involved in using a statistical software package to generate correct descriptive statistics including visual descriptive

statistics (histograms, box plots), measures of centre (mean, median, mode), measures of variation (standard deviation, range, interquartile range, coefficient of variation), and outliers. They all demonstrated instrumental understanding of terminologies of the measures of centre by accurately providing a definition of the mean, median and mode in their own words. They all also provided an accurate definition of at least one measure of variation, but no one provided accurate definitions of all measures of variation, that is, range, standard deviation, interquartile range, and coefficient of variation. Most of them provided an accurate definition of an outlier in their own words while few provided a definition of either a histogram or a box plot. Instead of definitions, most of them provided accurate descriptions of what key parts of the visual representation meant. For example, they accurately stated that the heights of the histogram referred to frequency and that the ends of the box in the box plot were the first and third quartiles.

Most participants demonstrated relational understanding of measures of centre, outliers, and visual descriptive statistics by correctly interpreting them in relation to a context. However, only few of them demonstrated this understanding of the measures of variation by correctly interpreting them. In particular, most participants did not appear to understand what the measure of variation would indicate about the nature of the data. When considering the descriptive statistics together to draw a conclusion about the problem, most participants were successful at arriving at a conclusion to address the problem presented in the task. Yet, most participants arrived at their conclusions by examining the measures of centre and variation, but few participants considered visual descriptive statistics when making a conclusion.

Most participants demonstrated relational understanding regarding making choices by choosing the appropriate measure of centre for their data and correctly explained their reasoning behind their choice. Most participants chose the appropriate measure of variation for their data. All participants provided correct but incomplete justifications of their choice. In particular, they justified their choice by comparing different measures of variation, but no participant considered all measures of variation. No participant stated which appropriate visual descriptive statistic was better for a situation. As only one method of finding outliers was presented to students in the course, it was not expected nor appropriate for students to make a choice and justify a choice for this concept.

No participant demonstrated relational understanding regarding underlying basis of the descriptive statistics concepts. For example, no participant explained why the mean, median or mode were measuring the centre of the data.

DISCUSSION AND CONCLUSION

The focus of this exploratory study was to determine the nature of students' understanding that seemed possible with a story-based task as the central learning activity. It was not to evaluate the intervention in terms of change to the students' knowledge at the beginning compared to the end of the course or otherwise. The students' work directly associated with the story-based task demonstrated that they were able to develop instrumental and relational understanding of descriptive statistics concepts to various degrees. Instrumental understanding involves being able to follow steps of algorithms or define terminologies and relational understanding involves the added elements of being able to interpret contexts, make choices or explain the underlying bases of the concepts. All/most (over 75%) of the participants demonstrated instrumental understanding for all four categories of concepts and relational understanding on two levels for three of the categories. However, many of them did not demonstrate relational understanding for measures of variation and all of them did not demonstrate it regarding the underlying basis (reason/why) for all four categories of concepts.

Some of these findings are similar to those of past research on students' learning of some of these concepts, thus providing further evidence of them. For example, Gougis et al. (2016) and Mathews and Clark (2007) also found that students had difficulty with relational understanding of variation. On the other hand, participants in this study did not demonstrate the same level of difficulty for some concepts as found in previous studies. For example, the participants were able to provide accurate definitions of measures of centre and at least one type of variation (e.g., standard deviation), which is different from findings by Lavy and Mashiach-Eizenberg (2009) and Mathews and Clark (2007) that indicated students struggled with even understanding the definition of standard deviation. Further, participants successfully interpreted a histogram in the context of the story, which is different from the

findings of Kaplan et al. (2014) who found that students had difficulty reading histograms. Thus the findings validate and extend what we know about students' knowledge or understanding of specific descriptive statistics concepts.

Studies on post-secondary students' learning of statistics have attended to students' knowledge or understanding at the beginning and/or end of an undergraduate statistics course but often do not specifically make connections to a theoretical perspective of understanding (e.g., Dubreil-Fremont et al., 2014). From our review of the literature, a few studies used *conceptual understanding* (e.g., Mathews & Clark, 2007; Verkoeijen, Imbos, van de Wiel, Berger, & Schmidt, 2002), but often, without defining it theoretically and at times it seemed to be used as a term to indicate knowledge of a concept. These studies focused on the strength (e.g., poor/weak), stage (e.g., low level, developing) or comparative state (e.g., no improvement) of students' understanding with more emphasis on students' difficulties or misconceptions (Lemieux & Chapman, 2020). The difference between such studies and our study is that we have a constructivist goal to identify and describe the understanding of statistics concepts students developed and demonstrated through the story-based tasks and not present a deficit perspective of their knowledge or performance. While conceptual understanding is similar to relational understanding, this study offers the combination of instrumental and relational understanding, two levels commonly used in research in mathematics education, as ways of making sense of where students are in their trajectory of learning the statistics concepts to support future learning.

To conclude, the findings suggest that a pedagogical intervention involving story-based tasks has the potential to make a positive impact on students' learning of descriptive statistics, particularly regarding students' understanding of concepts in areas that previous research suggests are difficult for students to learn. However, use of such tasks seems to be more successful in supporting instrumental understanding and some aspects of relational understanding of the concepts and least successful in supporting knowledge of why/meaning of the concept. As this is an initial exploratory study of the use of these tasks, the implication is that further study is needed to understand the type of pedagogical support that is needed to engage students in the story-based task in a way that fosters more depth in relational understanding. For example, providing students with experiences to understand what it means, and how, to engage in learning for relational understanding may be a necessary prerequisite to engaging in the story-based activities. As previously noted, this paper dealt with only one aspect of a larger project, so it also does not provide a full picture of the intervention. The project includes the use of different story-based tasks for other areas of statistics covered in the course and investigation of the impact of the intervention on students' beliefs of the usefulness of statistics, their perspective of learning through the story-based tasks, and elements of the tasks that support their understanding. As a case study, there are obvious limitations to the study regarding generalisations. The intent here is to highlight the potential of the story-based tasks, but further studies are needed regarding, for example, different types of story-based tasks with different demographics of students, other than business majors, to provide further evidence regarding such tasks for descriptive statistics.

REFERENCES

Chan, S. W., & Ismail, Z. (2013). Assessing misconceptions in reasoning about variability among high school students. *Procedia-Social and Behavioral Sciences, 93*, 1478-1483. https://doi.org/ 10.1016/j.sbspro.2013.10.067

Dubreil-Frémont, V., Chevallier-Gaté, C., & Zendrera, N. (2014). Students' conceptions of average and standard deviation. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics (C142, pp. 1-4)*. International Statistical Institute. http://iase-web.org/icots/9/proceedings/pdfs/ICOTS9_C142_DUBREILFREMONT.pdf?1405041821

Egan, K. (1986). *Teaching as story telling: An alternative approach to teaching and curriculum in the elementary school*. London, ON: The Althouse Press.

Garfield, J., Aliaga, M., Cobb, G. W., Cuff, C., Gould, R., Lock, R., Moore, T., Rossman, A., Stephenson, B, Utts, J., Velleman, P., & Witmer, J. (2005). *Guidelines for assessment and instruction in statistics education (GAISE): College report*. American Statistical Association. https://www.amstat.org/asa/files/pdfs/GAISE/2005GaiseCollege_Full.pdf

Gougis, R. D., Stomberg, J. F., O'Hare, A. T., O'Reilly, C. M., Bader, N. E., Meixner, T., & Carey, C. C. (2016). Post-secondary science students' explanations of *randomness* and *variation* and implications for science learning. *International Journal of Science and Mathematics Education, 15*(6), 1039-1056. https://doi.org/10.1007/s10763-016-9737-7

Kaplan, J. J., Gabrosek, J. G., Curtiss, P., & Malone, C. (2014). Investigating student understanding of histograms. *Journal of Statistics Educ*ation, 22(2), 1-30. https://doi.org/10.1080/10691898.2014.11889701

Lavy, I., & Mashiach-Eizenberg, M. (2009). The interplay between spoken language and informal definitions of statistical concepts. *Journal of Statistics Education, 17*(1), 1-9. https://doi.org/10.1080/10691898.2009.11889502

Lemieux, C. (2020). The use of story-based tasks in post-secondary students' learning of statistics. [Doctoral dissertation, University of Calgary]. PRISM. https://prism.ucalgary.ca/handle/1880/111565

Lemieux, C. & Chapman, O. (2020). Types of understanding of statistics demonstrated by postsecondary students. *The Mathematicians Educator*, *1*(1), 43-57.

http://math.nie.edu.sg/ame/journal/themathematicianeducator/tme/2020vol1no1/article4.pdf *(previously The Mathematics Educator)*

Mathews, D. & Clark, J. M. (2007). *Successful students' conceptions of mean, standard deviation, and the central limit theorem*. Retrieved from https://www.researchgate.net/publication/253438034_Successful_Students'_Conceptions_of_Mean_Standard_Deviation_and_The_Central_Limit_Theorem

Ramirez, C., Schau, C., & Emmioglu, E. (2012). The importance of attitudes in statistics education. *Statistics Education Research Journal, 11*(2), 57-71. https://iase-web.org/documents/SERJ/SERJ11(2)_Ramirez.pdf

Roberts, N., & Stylianides, A. J. (2013). Telling and illustrating stories of parity: A classroom-based design experiment on young children's use of narrative in mathematics. *ZDM – Mathematics Education, 45*(3), 453-467. https://doi.org/10.1007/s11858-012-0474-2

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Arithmetic Teacher, 77*, 20-26.

Stake, R. E. (2005). Qualitative case studies. In N. K. Denzin & Y. S. Lincoln (Eds.), *The Sage handbook of qualitative research* (3rd ed., pp. 443-466). Thousand Oaks, CA: Sage Publications.

Tishkovskaya, S., & Lancaster, G. A. (2012). Statistical education in the 21st century: A review of challenges, teaching innovations, and strategies for reform. *Journal of Statistics Education, 20*(2), 1-56. https://doi.org/10.1080/10691898.2012.11889641

Verkoeijen, P. P. J. L., Imbos, Tj., van de Wiel, M. W. J., Berger, M. P. F., & Schmidt, H. G. (2002). Assessing knowledge structures in a constructive statistical learning environment. *Journal of Statistics Education, 10*(2). https://jse.amstat.org/v10n2/verkoeijen.html

Zaidan, A., Ismail, Z., Yusof, Y. M., & Kashefi, H. (2012). Misconceptions in descriptive statistics among postgraduates in social sciences. *Procedia - Social and Behavioral Sciences, 46*, 3535 – 3540. https://doi.org/10.1016/j.sbspro.2012.06.100

Zazkis, R., & Liljedahl, P. (2009). *Teaching mathematics as storytelling*. Rotterdam, The Netherlands: Sense Publishers.