Diagnostic tests in a first year Mathematics subject

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Introduction

In this paper, we explore the issues around diagnostic testing of first year students in mathematics and give an example of the use of such a test in a large first year subject. We compare the scores on the diagnostic test with the end of semester results for 2005. Opinions and insights from students and staff were also collected using open-ended surveys.

Background

Failure rates in first year mathematics subjects are higher than in other discipline areas and mathematics departments in Australia and New Zealand are under some pressure to lower those rates. One of the reasons for the high failure rate that has been cited is the differing mathematical backgrounds of students who enter university. Bridging programs are common throughout the world and may consist of skills and knowledge or may be more of an introduction to the mathematical thinking required at university (Wood 2001). Universities may offer mathematics subjects at different levels depending on the mathematical background of the students. For example, the University of Sydney offers mathematics at three levels (Britton, Daners and Stewart 2006). But how do you ensure that the students enrol in the level appropriate to their background, skills and motivation?

Internationally there have been differing responses to this situation. In most European countries, matriculation from high school is assumed to be sufficient for university study and mathematics departments have not been too worried if the failure rate is high, though this is beginning to change. For example, the University of Vienna has completion rates of about 25% for their mathematics degrees (Ulovec 2006). Recently, the Austrian government has changed the funding formula from enrolment to completion so the mathematics department is becoming more sensitive to drop out rates and has instituted a bridging course to assist with the transition to university. The results of this intervention included a decrease in drop out rates and an increase in pass rates.

In South Africa, there is a different problem with the transition to mathematics at university. Students from disadvantaged backgrounds do not perform academically well at school due to historical under-resourcing of their schools. South African universities have instituted a national test which aims to assess the potential of students to learn mathematics rather than their current knowledge and skills (Frith, Frith and Conradie 2006). The students are given written instructional materials about unfamiliar mathematical content, interspersed with questions which quiz the students' ability to learn from the instruction, work with definitions, make valid deductions and generalise. The test seems to work in that these students, after an initial transition period at university, do as well as other students who gained entry to university mathematics through normal school selection criteria.

In Australia all students have access to reasonably equitable schooling and have taken examinations that demonstrate their achievements in mathematics. There is, however, still a large variation in the ability of these students to cope with first year mathematics. At The University of Sydney, Britton, Daners and Stewart (2006) investigated the use of a diagnostic test as a predictor for success in their advanced level first year mathematics subject. Their analysis indicated that the results from the test together with the results from their school examinations were a good predictor for success in the advanced units. Their aim is to correctly identify students who will be able to handle the advanced units in order to reduce the drop down rate which is disruptive to the students and to the staffing of tutorials.

At the other end of the spectrum, diagnostic tests are used to identify students who are at risk of failure and to recommend remediation or support programs as at Queensland University of Technology (Coutis, Cuthbert and MacGillivray 2002). Students who fail the diagnostic test and who use the support programs are more likely to succeed than those who do not seek help but this could be a measure of their motivation as well as the efficacy of the remedial program. Other studies have found that for adult learners returning to study diagnostic tests are not good predictors of success at university.

Optional diagnostic testing does not seem to be successful. Online tests available for students are rarely done unless it is required by the program as found by Britton, Daners and Stewart (2006) and noted elsewhere.

Diagnostic testing, whether online or given in class, uses resources including the time of staff and students. Is the diagnostic testing procedure worthwhile or would we be better off just suggesting that students who felt under-prepared take remedial action?

The context

The University of Technology, Sydney has an Engineering program that attracts a diverse group of students from extensively different backgrounds. The majority of students (80%) are high-school leavers and the remainder are students coming from TAFE or from the workforce. There are many more male students (88%) than females as is common in engineering courses. The cultural background of students is very diverse. There were more non-English background speakers (61%) than English speakers (38%).

Because of this diverse background the Department of Mathematical Sciences at UTS has devised a pathway where students can take a subject in foundation mathematics for credit as an elective in their degree program. Students are able to take this subject in semester 1, do the first engineering mathematics subject (Mathematical Modelling 1) in semester 2, the second engineering mathematics subject over the summer and be on par with their peer group by the start of the second year. The Engineering Faculty is extremely happy with the arrangement as students have an opportunity to reach the required skills by the beginning of their second year. We should note that not all engineering programs allow this as some have no electives. In this case students are advised to take the foundation subject without credit.

Mathematical Modelling 1 has a class size of around 600 students in the Autumn semester and so it is not practical to devote time to each student in order to determine if they can succeed in the subject. It is more practical to test the students on what the lecturer believes to be the basic skills required for the course. Based upon the results of this test it is recommended of some students to change to the foundation subject. There is no compulsion here to change subject and many students choose to continue in Mathematical Modelling 1, against recommendation.

Preparing the test

The content of the engineering mathematics subject is a standard entry level Calculus course. Although no topic is exceptionally difficult, taken as a whole, this is a demanding subject for the average first year student. The mathematical assumed knowledge for the subject is NSW Extension 1 mathematics. The diagnostic test is not designed to test if they still know all the Extension 1 material. Instead it is designed to determine if a student has the required background to complete the course seeing the material for a first time. It examines elementary algebra, curve sketching, simple differentiation and integration (normally of polynomials). The test consists of 20 multiple choice questions and is administered during one of the lectures in the first week.

To give you a flavour of the test, some sample questions are:

1. The expression $27^{-\frac{1}{3}}$ can be written as:

(a) $\frac{1}{3}$ (c)

(b) $\frac{1}{9}$ (d)

5. The expression $3x^2 - 7x + 2$ can be factorised as

(a) $(3x+1)(x-2)$ (c) $(3x+1)(x+2)$

(b) $(3x-1)(x-2)$ (d) $(3x-1)(x+2)$

9. The straight line with equation $x - 4y - 2 = 0$ has gradient

(a) 4 (c) $\frac{1}{4}$

(b) -2 (d) $\frac{1}{2}$

12. If $y = \sin^2(3x-2)$ then the derivative $\frac{dy}{dx}$ is:

(a) $2\sin(3x-2)$ (c) $2\cos(3x-2)$

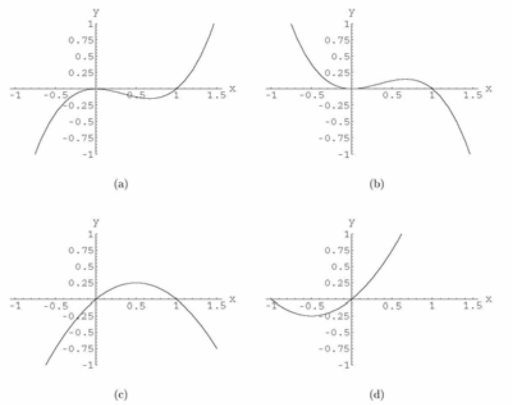
(b) $6\sin(3x-2)\cos(3x-2)$ (d) $-6\sin(3x-2)\cos(3x -$

17. If $\sin\theta = \frac{1}{\sqrt{2}}$ and $\cos\theta = -\frac{1}{\sqrt{2}}$ for $0 \le \theta \le 2\pi$, then

(a) $\theta = \frac{3\pi}{4}$ (c) $\theta =$

(b) $\theta = \frac{\pi}{4}$ (d) $\theta =$

19. The graph of $f(x) = x^3 - x^2$ has the form:



Advice to students

Once the results have been tabulated, the marks are entered into the online gradebook where students are able to view the mark they have received for the test. It is expected that students who have the required mathematical background should be able to score at least 70% in this test. Students who scored between 50-69% are advised to do some revision or extra study in order to have a sufficient level of assumed knowledge. Students scoring at the lower end of this interval are advised to think about dropping back to the foundation subject to build up their knowledge. Students who scored less than 50% are strongly encouraged to drop back to the foundation subject and brush up or learn the essentials. Of course this test only served as a guide to students so that they could evaluate their own knowledge.

Results

Quantitative data

The data are from the first semester 2005. Performing a simple scatter plot with the diagnostic test on the bottom axes and the final on the side.

Figure 1 is a simple scatter plot with the results of the diagnostic test on the x axis and the results of the final examination on the y axis. The data displayed are from the first semester 2005.

The results from the scatter plot showed a significant positive correlation of 0.59, which is promising. Note that the data used does not include all students who sat the diagnostic test, as there were 30 students who changed to the foundation subject and some students who dropped out of the university prior to subject completion.

Of those who completed the diagnostic test and the examination for the mainstream mathematics subject, we have the recommendation breakdown that the students received.

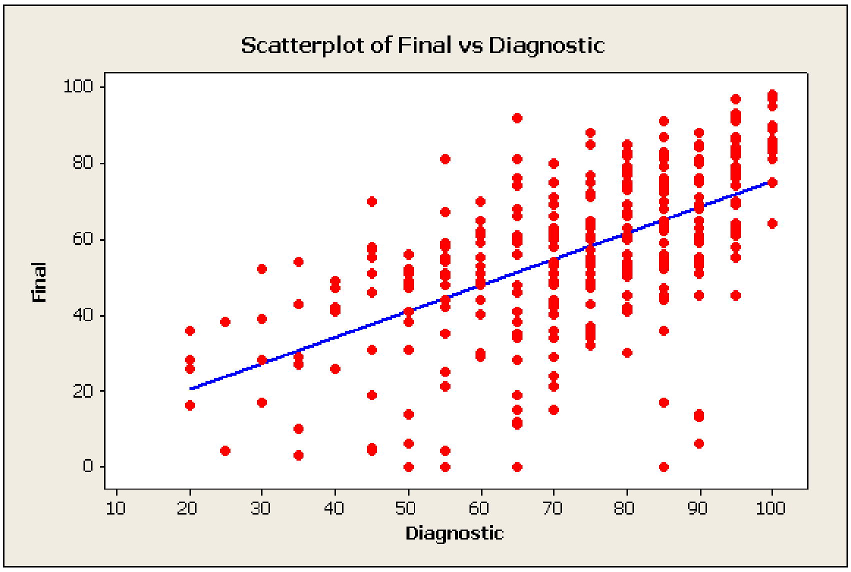


Figure 1. Scatterplot of final result vs diagnostic test mark

Table 1. Number of students in each cross category

		Diagnostic Test		
		0-49	50-69	70-100
Final Exam	50-100	8	38	191
	40-49	6	16	23
	0-39	18	21	20

From the results in the first column of Table 1, it is clear that students who got less than 50% in the diagnostic test were given the right advise as only one quarter (8 out of 32) of the students, who went against the recommendation to drop back to the foundation subject, passed Mathematical Modelling 1. Approximately half of the students who scored in the 50-69% range of the diagnostic test and stayed in the subject Mathematical Modelling 1, failed. Students who scored above 70% in the diagnostic test performed consistently better in the final examination than students in the other categories, with only 18% of these students failing. It is likely that there were other reasons, or influencing factors, for this 18% failure.

It seems clear that more students should have done the foundation mathematical subject. From Table 1 it is possible to see that the students scoring between 50-69% in the diagnostic test have equal chances of passing or failing the subject. We break down this category even further in Table 2.

Table 2. Number of students scoring 50-65

Final Exam	Diagnostic Test 50	55	60	65
50-100	4	12	10	11
40-49	6	3	4	3
0-39	5	5	2	9

There are two interesting observations that arise from Table 2. The first being that of the students who scored 50% in the diagnostic test, less then a third of them had passed. The other is that a large number of students who scored 65% on the diagnostic test, ended up scoring less than 40% as their final grade. The students that did score 50% should have been encouraged to do the foundation subject.

It appears that the diagnostic test is a better indication of the students who will struggle to pass Mathematical Modelling 1, rather than predicting those who will pass. Passing the diagnostic test does not mean that they will do well in the subject, as the results showed a large variability in their grades.

Qualitative data

The students were given a survey with open-ended questions in the last lecture of semester. The purpose of the survey was to gauge their feelings about the diagnostic test they had taken at the start of semester and whether it had an impact on their learning during the semester. Academic staff involved in the teaching first year Mathematics were asked what they believe is the importance of diagnostic testing and their replies are summarised below. The interesting fact here is the divergence between the replies of the students and those of the staff. The staff were considering practical issues such as weeding out weaker students whereas the students had an emotional response to the testing. If they did well it gave them confidence and if they did badly many took this as an opportunity, not to do the lower level subject, but to work harder.

Staff

When academic staff were asked why diagnostic testing was important and the reasons given were:

- to avoid high failure rates;
- to provide students with weaker backgrounds with a pathway into successful mathematics study at university;
- to cover ourselves in case of high failure rates (that is, we tested the students and advised them to do a lower level but they did not follow our advice and failed, therefore it is not our fault);
- to obtain data on the knowledge and skills of incoming students; and
- to inform our teaching.

Students

One surprising result of this survey was that many of the students who passed the test gained confidence from this. The reasons for this varied from “getting a sense of being in the right place” and that they “could handle this course”.

- I felt really confident and fitted in well, understanding the basics enabled me to broaden my understanding on this topic.
- Happy it allowed me to start the subject with confidence.
- It gave me confidence to go through the subject.
- It allowed you to make the choice between Foundation and MM1.
- It helped me to identify my weakness.

- It gives people a insight as to how much work/extra work they will need to do to keep up in the subject.
- No good for Non recent school leavers. Not doing maths in a while doesn’t help. Not using a calculator even though I did the bridging course.
- It had no purpose as I had a long break and my mind was not ready and alert.

When shown the student responses, the current lecturer said: “I always thought that the diagnostic test was there to weed out the students who were not ready for the subject. I was not aware that those who did well were getting confidence from it as well!”

The students who did change to foundation mathematics found it to be a positive experience. Here are some of their comments about their experiences.

- I failed the diagnostic test miserably, which prompted me to pick this subject up [change to foundation mathematics], because I knew I didn't have the knowledge for MM1.
- It was the first test we did that make me realise I should do foundation maths.
- Because I wouldn't have known what level of maths was in maths Mod 1.
- It was the increasing difficulty of Math Modelling not my test result that convinced me to change to Foundation.
- As Foundation Mathematics help you build on basics. It gave me the confidence on algebra and calculus to now move to Math Modelling 1.

Conclusion

The correlation between results in the diagnostic test and the final examination is reasonably high but does not account for all the variation. It is a better predictor for those who failed the diagnostic test as most went on to fail the subject. Of those who passed the diagnostic test, the results are less useful and other factors may be important. For example, the test is not a predictor of motivation and work ethics.

The open-ended surveys gave valuable insight into the emotional states of the students in relation to the diagnostic test and their perceptions of mathematics at university. It was surprising that the main themes from the survey responses were to do with self confidence gained and the opportunity of a challenge to work harder and pass. There were the odd negative comments – particularly those who would have liked to be warned so that they could have prepared for the test. Those who moved to the lower level foundation subject were positive about their experience and felt more confident with their ability to work with mathematics at university. The pass rate in the foundation subject was very high.
It is clear that failure in the diagnostic test led to a high chance of failure in the subject and that those students who changed to the foundation subject found it a positive experience. We need to encourage more students to be realistic about their mathematical ability and background so that they are fully prepared for mathematics at university. Tackling a mathematics intensive degree like engineering will always be a struggle for students with weak backgrounds.

There are various alternative options here.

- We can run an optional online, computer marked diagnostic test, however most students do not seem to do these.
- We can give general advice and hope that the correct students will change to the foundation subject. We may still have under-prepared students and high failure rates.
- We can improve the diagnostic test. Indeed Britton, Daners and Stewart (2006) found that three questions were sufficient to predict success in their advanced mathematics subject.

Considering the qualitative comments in particular, we believe that the diagnostic test was useful in alerting those students who were seriously under prepared for mathematics at university. A small amount of pain for the students in their first lecture can lead to a positive outcome.

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