Decision analysis under uncertainty for e-learning environment

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Abstract: *In this paper, we discuss the notions of fuzzy sets and fuzzy IC-Bags as well as propose a soft decision analysis methodology for analysing student performance measures and investigate its applicability within the framework of e-learning where, in the absence of face-to-face interaction, often it is necessary to rely considerably on the frequencies and characteristics of interactions of the learners with a pre-defined set of learning objects included in the process.*

Introduction

In case of e-learning, people learn through accessing and interacting with well-designed information sources which can improve the quality of the attributes of their performance. In case of analysing student performance measures, we may come across the avenues of accessing linguistic abilities, mathematical abilities, spatial intelligence as well as interpersonal communication skills. For e-learning, the judgement patterns are not just paper-based, but they are often based on the degree, type and lengths of interactions of the learners with a pre-defined set of multimedia learning objects included in the process. This set can include the sample set of solved problems, case-studies, simulated scenarios etc. For determining the optimal levels of interactions, we consider a soft decision analysis scenario where we depend on crisp limit sets as determined by a panel of experts in the concerned area. These interval-valued limit sets can incorporate psychological hesitation factors (Chakrabarty 2000b) in the form of bias which needs to be quantified in order to maintain the consistency of the judgement process. These psychological hesitation factors can relate to the Myers-Briggs personality types (Mallach 2000) corresponding to the characteristics of the individual experts of the panel, and can influence the nature and type of intervals set by them regarding the optimal frequencies of interaction required for attaining the related set of learning outcomes involving tacit knowledge in the form of processed information.

In an attempt to identify the vaguely defined set of variables contributing to these patterns, we observe that in case of traditional learning processes, the learners often receive the information without being involved with much interactive procedures. Bowles (2004, p89-90) said

> Traditional learning processes tend to reduce learners to the status of passive recipients of information from on high, while evaluating 'achievement' through simplistic, standardised tests of knowledge retention and performance skills. Both these characteristics have a tendency to limit the types of knowledge that learning engenders. By contrast, e-learning that deploys learner centred pedagogies and continuous self-evaluation can be used to transmit a far wider range of skills and knowledge while remaining relevant to organisational needs.

For each pre-determined learning objective, it is necessary to determine the optimal range of the frequency of interaction that the learner should have with the elements of the pre-defined set of learning objects. In case of e-learning, the elements of this set are often found to be the multimedia objects. For a formal assessment of performance measures, we request each member of a panel of experts to assign an interval value for each element of the Cartesian product of the set of the pre-determined learning objectives and the set of pre-defined learning objects. We also determine the possibilistic degree of importance of each learning outcome as conceptualised by individual members of the panel. The attributes of the concerned e-knowledge such as qualitative and quantitative analytic capabilities can influence the formalism of the e-learning objectives. Ambiguity and hesitation are the two perceived forms of uncertainty that play crucial roles in case of assessing soft performance measures strongly influenced by human judgement patterns which relates to the criteria set by the individual members of the panel. It originates from the fact that whenever human experts attempt to judge the analytic capabilities of a learner based on the patterns of his/her interactions with the pre-defined set of learning objects, they are never hundred percent confident about their decisions. Evidently, the experts' decisions regarding the performance measures incorporate ambiguity which amalgamates with the cognitive hesitation patterns. The confidence factors in these cases are always found to be soft and relates to the intuition of the human experts.

For assessing e-learning performance measures, the interactions between the number of controllable and uncontrollable variables can give rise to a range of psychological hesitation patterns. Most commonly, this is caused while incorporating mixed mode delivery involving a set of interactive multimedia objects. The types of these patterns are highly contextual in nature and can be found to be deeply embedded in the procedural logic underlying the process model. Factors that are incorporated into the uncertainty modelling algorithms involving different types are addressed in this paper and the notion of fuzzy IC-Bags (Chakrabarty 2004a) is used as a tool for the purpose of analysing the concerned patterns.

Fuzzy sets, by exploiting the degrees of memberships, are capable to express gradual transitions from membership to nonmembership (Klir 2000). This notion, with its multitude of applications in the areas of artificial intelligence, can provide a strong alternative to the traditional concept of set membership and consequently the degrees of membership of the elements can play essential roles in the explanation and quantification of uncertainty. The area of psycholinguistics is found to be essential in order to study the connection between the human use of linguistic terms in different contexts with the associated fuzzy sets and operations on the fuzzy sets (Klir 2000). The pseudo-contradictory nature of the variables in the decision space can play an interesting role while observing the above mentioned semi-deterministic and non-crisp patterns. In this context, the possibilistic degrees of importance of each learning outcome as conceptualised by individual members of the panel of experts are found to be extremely useful for the decision modelling related to the assessment and analysis of the e-learning performance measures.

Fuzzy sets and fuzzy IC-bags

Let X be a classical set of objects called the universe and let x be any arbitrary element of X. Membership in a classical subset A of X is often viewed by a characteristic function μ_A from X to $\{1.0\}$ such that

$$\mu_A(x) = \begin{cases} 1 \; iff \;\; x \in A, \\ 0 \; iff \;\; x \notin \mathrm{A}, \end{cases}$$

where $\{0,1\}$ is called a valuation set. If the valuation set is allowed to be the real interval $[0.1]$, then A is called a fuzzy set (Zadeh 1965) and $\mu_A(x)$ is called the grade of membership of x in A. The closer the value of $\mu_A(x)$ is to 1, the more x belongs to A. Thus, we can view A as a subset of X that has no sharp boundary. $\mu_A(x)$ can be interpreted as a *degree of compatibility* of the predicate associated with A and the object x or the degree of possibility that a phenomenon x is restricted to A.

Bag structure, as introduced in (Yager 1986) is found to be an extremely important concept in case of describing the collections where the redundancy of objects present in the collection are to be taken care of. Hence, for the relational data models with multiset semantics, the concept of bags is being successfully used.

A bag (or crisp bag) B drawn from a set X is represented by a function $Count_B$ or C_B defined as $C_B : X \to N$ where N represents the set of non-negative integers. In (Chakrabarty 2000b), the author used the notion of fuzzy bags for a decision analysis problem regarding student evaluation involving hesitation in grading for multi-individual type of decision support systems.

In (Chakrabarty 2000a), the author introduced the notion of IC-Bags as below:
Let Ω be any non empty set. Then an IC bag β drawn from Ω is characterized by a pair of functions C_l^β and C_u^β such that

$C_l^\beta : \Omega \to N$ and $C_u^\beta : \Omega \to N$

and $C_l^\beta(x) \le C_u^\beta(x)\ \forall x \in \Omega$, where N represents the set of non-negative integers. If $\Omega = (x_1, x_2, \ldots, x_n)$, then an IC bag β drawn from Ω is represented as

$\beta = \{x_i /(C_l^\beta(x_i), C_u^\beta(x_i))\}$

where $i = 1,2,\ldots\ldots..n.$ We call $C_l^\beta(x_i)$ the minimum count for x_i in the IC bag β and $C_u^\beta(x_i)$ the maximum count for x_i in the IC bag β.

In (Chakrabarty 2004b), the author discussed about selection from IC-Bags, Semi-Structured specifications with respect to IC-Bag based systems involving decision analysis model involving relational data entries for contextual rule importance grades and associated hesitation factors.

In (Chakrabarty 2004a), author defined and characterised the notion Fuzzy IC-Bags as follows:

A Fuzzy IC-Bag ϕ drawn from a non-empty set $\Re$ is characterised by the function ξ^ϕ such that $\xi^\phi : \Re \to U_I$ where U_I represents the set of all IC-Bags drawn from the continuum $I = [0,1]$. Thus for any $\omega \in \Re, \xi^\phi(\omega)$ is an IC-Bag drawn from I. But since any IC-Bag can itself be characterised by a pair of functions over its set, hence $\xi^\phi(\omega)$ can be characterised by the pair of functions

$C_l^{\xi\phi(\omega)} : I \to N$
$C_u^{\xi\phi(\omega)} : I \to N$

where $C_l^{\xi\phi(\omega)}(\alpha) \le C_u^{\xi\phi(\omega)}(\alpha)\ \forall\ \alpha \in I, N$ being the set of non-negative integers.

Fuzzy IC-bags in soft decision analysis under the framework of e-learning

The degree of interaction of the learners with a pre-defined set of learning objects, including multimedia objects, in case of e-learning can be conceived as a controllable variable which depends on the personality type, cognitive capabilities and motivational variations of a learner. The attributes of the concerned knowledge can be regarded as a set of uncontrollable variables in the part of the learner which will infuence, and in turn will be influenced by the psychological formalism, cognitive preferences, and often the contextual hesitation factor. Similarly, the hesitation factor, in the form of bias, can influence the deterministic patterns while each member of a panel of experts assigns an interval value for each element of the Cartesian product of the set of the pre-determined learning objectives and the set of pre-defined learning objects. From the collective decision of the experts, we determine the optimal range of the frequency of interaction that the learner should have with the elements of the pre-defined set of learning objects. For a formal assessment of performance measures, the assigned interval value for each element of the Cartesian product of the set of the pre-determined learning objectives and the set of pre-defined learning objects is considered. Further, the possibilistic degree of importance of each learning outcome as conceptualised by individual members of the panel is determined. The attributes of the concerned e-knowledge such as qualitative and quantitative analytic capabilities are found to influence the formalism of the e-learning objectives. It is important these influences be analysed while quantifying the overall performance measures.

While analysing student performance measures in terms of the degree of interaction, we often come across uncertainty and non-crisp situations. In case of a group support system, when experts try to decide about the required frequency of interaction of the learner with the respective learning objects, the hesitation factors embedded with their input measures should be identified for developing an appropriate soft methodology for obtaining the optimal feasible solution.

We consider a given set of experts $E = \{E_1, E_2, \dots E_n\}$ and their input measures with respect to a set $L = \{L_1, L_2, \dots L_m\}$ of learning outcomes involving tacit knowledge. The input measures can be represented by

$$\underline{M}^i : L \to N \quad \text{and} \quad \overline{M^i} : L \to N$$

where $\underline{M}^i(x) \le \overline{M^i}(x)\ \forall x \in L;\ L = \{L_1, L_2, \dots L_m\},\ i = 1,2,\dots n.$ The psychological hesitation factor associated with the deterministic input measures for each E_i can be given by $\phi_j(x)$ representing the difference between $\overline{M^i}(x)$ and $\underline{M}^i(x)$ for each $x \in L$.

Individual hesitation factors can be influenced by the possibilistic degree of importance of each learning outcome as conceptualised by the individual expert. For each expert E_i we define a fuzzy membership function $f^i : L \to [0,1]$ and the decision space for each learning outcome $x \in L$ can be represented by each E_i as $\{\underline{M}^i(x), \overline{M}^i(x), f^i(x)\}$.

Thus, for each learning outcome $x \in L$ in the concerned knowledge base, the psychological bias factor $\gamma^i(x)$ occurring with respect to the specific hesitation pattern of the expert E_i can be represented by

$$\gamma^i(x) = \frac{1}{i}\sum_{i=1}^{n} f^i(x) - f^i(x)$$

So, the indeterministic perceptual factor of the frequency of interaction as determined by E_i can be obtained by

$$\eta_i(x) = \frac{1}{2}.\gamma^i(x)(\overline{M}^i(x) + \underline{M}^i(x))$$

The optimal deterministic frequency of interaction $\psi_i(x)$ due to E_i for each $x \in L$ can be obtained by

$$\psi_i(x) = \frac{1}{2}\{\overline{M^i}(x) + \underline{M}^i(x)\} + \eta_i(x).$$

The final optimal deterministic frequency of interaction as perceived by E can be obtained by $\sum_{i=1}^{n} \psi_i(x).$

Conclusion

It is observed that in case of e-learning, people learn through accessing and interacting with well-designed information sources which can improve the quality of the attributes of their performance. In this paper, we have considered a soft decision analysis framework for formalising the effects of the indeterministic factors occurring with the judgment patterns in the context of e-learning where a panel of experts is considered where each member of the panel individually express their decisions regarding the optimal frequencies of interaction that the learner should ideally have with the elements of the pre-defined set of learning objects. In the absence of face-to-face interaction, the elements of this set are often found to be the multimedia objects and in order to have a formal assessment of the required performance measures, we request each member the panel to assign a interval value for each element of the Cartesian product of the set of the pre-determined learning objectives and the set of pre-defined learning objects. We further determine the possibilistic degree of importance of each learning outcome as conceptualised by individual members of the panel. The concerned judgement patterns are often based on the degree, type and lengths of interactions of the learners with a pre-defined set of multimedia learning objects and can occur in the design phase of the decision making with approximate, imprecise description of the objects in the form of learning objectives. In such cases, we often have to depend on crisp limit sets as determined by members of the panel. The interval-valued limit sets are considered and the final optimal deterministic frequency of interaction as perceived by the panel is obtained.

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