describing this line is to illustrate its derivation, with a section calculation in detail, on the principles above-described. The information required is the leverage l, or the distance from the toe of the section to the centre of combined action of gravity and cohesion.

Section of Figure B, 45 to 50 feet (Figure D). Try s = 3·3, 3·3 + 23·7 = 27·0 = b, width of base (50 feet),

		pounds
$\frac{23\cdot7 + 27\cdot0}{2} \times 5 \times 140$ = weight of section ..		17,745
Weight of masonry to 45 feet		71,505
	w =	89,250
27 × 3,600 value for cohesion at 50ft.,	C =	97,200
	W =	186,450

$$\text{Leverage} = \frac{3b^2 - s^2}{6b - 3s} = \frac{3\ (27)^2 - (3\cdot3)^2}{6 \times 27 - 3 \times 3\cdot3} = 14\cdot307 \text{ for Section C of G}$$

16·4083 from toe at 45 feet to centre of gravity.
3 3

19·7083	from toe at 50 feet to above point	71,505	= w
14·307	average of Section for	17,745	
5·4013	= difference. Weight to 50 feet =	89,250	

Taking the moments for the centre of action of these two gravities—

$$17{,}745\ Y = 71{,}505\ (5\cdot4013 - Y),$$

$$\frac{5\ 4013 \times 71.505}{89{,}250} = 4\ 3274 = Y.$$

$$14\cdot307$$

Centre of Gravity of weight to 50ft. = 18·6344 from toe of 50ft.

18·6344		C of G for 89,250 weight to 50 feet	= w.
13 5	$= \frac{b}{2}$	leverage for 97,200 cohesion	= C.
5·1344	= difference.	186,450 = W.	

Taking the moments for the centre of action of the two forces,

$$97{,}200\ x = 89{,}250\ (5{\cdot}1344 - x),$$

$$\frac{5{\cdot}1344 \times 89{,}250}{186{,}450} = 2{\cdot}4577 = x.$$

$$\frac{13{\cdot}5}{} \qquad \frac{b}{2}$$

$$P = \frac{62{\cdot}4\ H^2}{2} = 78{,}000 \qquad 15{\cdot}9577 = 1$$

$$L = \frac{50}{3} = 16{\cdot}\dot{6}$$

$$PL = 1{,}300{,}000,$$

$$186{,}450 : 78{,}000 = 16{\cdot}\dot{6} : \quad 6{\cdot}9724$$

$8{\cdot}9853$ Resultant inside Toe.

$$b = 27 \qquad \frac{b}{3} \qquad 9{\cdot}0000$$

$0{\cdot}0147$ Resultant outside M/3.

or practically at M/3.

27 — 18·63 = 8·37, C of G of mass from heel of wall.
18·63 — 8·99 = 9·64, Distance between lines reservoir full and reservoir empty.
Resultant is practically on the middle third of the base, and centre of gravity is less than 8 inches outside the middle third.

Upon the principle of resistance claimed, that is mass alone without allowance for cohesion, the Wegmann wall of Figure C has a factor of safety against overturning moment of a shade over 2 (2·05). It contains 4,752 square inches of base area per foot run of wall, and the horizontal sliding force is 78,000, so the stress to induce sliding is about 16½ pounds per square inch. The friction at two-thirds co-efficient for masonry is $\frac{2 \times 120{,}992}{3} \div 33 \times 144 = 17$ pounds per square inch; and since there is no cohesion, or since cohesion is not permitted as an asset, the sliding force and the frictional resistance to it are just about equal, and the wall has only a factor of 1 in this respect. It is true that Wegmann stipulates that sufficient frictional resistance to prevent sliding upon each horizontal plane is assumed to be provided; but if the wall be built of concrete, and the concrete has no cohesive strength or resisting property, how is this frictional resistance to be obtained? Hence, as said before, even in the Wegmann section the cohesive property is essential to supply resistance to sliding to safe conditions; and if to sliding or shear, why not to tensile stress, in which the cohesive strength is greater, and if not more reliable, at least more is known concerning it? Again, if cohesion must be called in to assist, as it were, unofficially, is it not better to call in that assistance openly and attach a definite value to it in calculations? If the estimated values prove to be incorrect, there is a better chance of the error being detected and remedied under those circumstances than is the case when the assistance is treated as if it were a beneficent fairy of uncertain attributes.

EMBANKMENT RETAINING WALLS.

Ordinary retaining walls for the support of embankments can only be treated upon in a general manner, since the conditions under which they are built vary continually and greatly. One wall may support a background of rock of S.G. 2·5 or more upon inclined greasy beds, which, on exposure to weather or to vibration, lose hold, when the mass may slip with momentum much in excess of hydrostatic pressure. Another may be backed with coarse lumps or root-interlaced earth, with natural slope of 1 to 1 or steeper; the backing may be dry sand or wet mud, with hydrostatic properties. Hence it is only possible to sketch the proposition generally.

With average backing, a vertical back of a wall might be assisted against overturning by friction from the backing acting against the tendency of the pressed side to lift, but such friction may be neglected, for it will be trifling in comparison with the cohesive strength of the mortar, and cannot take effect until the latter is destroyed, when it would be useless. The weight of the backing may be made use of by stepping the back of the wall in rectangular off-sets, or even by building the batter at the back.

In the following paragraphs the backing is assumed to be dry sand against a vertical back of the retaining wall. The natural slope, or the slope at which the sand will remain at rest, is taken as $1\frac{1}{2}$ horizontal to 1 vertical.

SURCHARGED WALLS.

It is convenient to consider the conditions of surcharge first, for, as will appear, the backing of a wall that is not surcharged is merely a special case of the surcharge condition with the surcharge removed.

Let a b, Figure I., represent a wall, say 12 feet high, supporting an embankment of sand, earth, etc., abde. Let bc represent the slope of repose of the material—which would be about $1\frac{1}{2}$ to 1, if the backing were loose sand—of which the angle cbd is α, or about $33^\circ\ 42'$ for sand. The line ae is the surcharge, and is, of course, parallel to bc (though it might be flatter, but that would merely diminish the effect). The angle bae is then $(90^\circ + \alpha)$, or $123^\circ\ 42'$ for sand.

The portion of this backing represented by cbd exerts no pressure on the wall, and has no effect other than that of supporting the mass above it. The whole of the thrust on the wall is therefore confined to that which is imparted by the mass lying between ae and bc, and for convenience, that mass is regarded as having a dimension perpendicular to the plane of the figure, of unity, or say one foot if the calculations are in feet.

Since ae and bc are parallel, the mass between them is, or may be, infinite in bulk. Cut off any portion of it, abt, with the line bt, and let the weight of the backing or material in abt be k pounds per cubic foot (say 100 for sand). Let the angle abt be represented by β. Angle bat $= (90 + \alpha)$; angle atb $= \{90 - (\alpha + \beta)\}$; let ab $=$ h. Then, at $= \dfrac{h \sin \beta}{\sin 90 - (\alpha + \beta)} =$

$$\frac{h \sin \beta}{\cos (\alpha + \beta)}$$

Bisect bt in j; join ja and lay off jg, $= \dfrac{ja}{3}$

Then g is the centre of gravity of the triangle.

Let m $=$ the weight of the mass abt, which is one unit thick, then,

$$m = \frac{k\,h \dfrac{h \sin \beta}{\operatorname{Cos} (\alpha + \beta)} \sin (90 + \alpha)}{2} = \frac{\dfrac{k\,h^2 \sin \beta}{\cos (\alpha + \beta)} \cos \alpha}{2}$$

$$m = \frac{k\,h^2 \cos \alpha \sin \beta}{2 \cos (\alpha + \beta)}$$

All the forces acting may be regarded as acting at the centre of gravity g. See Figure 2. The mass m, regarded as loose grains of the backing material, in its endeavour to obey the law of gravity in the direction mg, is met by the resistance of friction, which tends to arrange each set of grains at the inclination α. It is generally accepted that a co-efficient of friction, multiplied by the weight of the mass, represents the nett effect of the friction irrespective of the shape of the mass, or as shown in Figure 3, in which m is the mass and α the angle of inclination; the friction f $=$ m sin α, and sin α is the co-efficient of friction. The force f, or m sin α, acts parallel to the angle of inclination, which in the case now under consideration is the angle of repose. The co-efficient of friction is therefore constant through all the planes of every wedge or triangle, and m sin α is constant throughout.

To return to Figure 2. At the point g the mass or force m meets the force f. which is m sin α, and the resultant of these two forces, which are in partial opposition, is mf in the force diagram.

$$(mf) = m \cos \alpha.$$

The resultant mf, or m cos α, is met by the reactionary supporting force s, of the base of the wedge under consideration—

for the wedge is regarded as acting alone, and therefore sliding on, or partly supported by this base. The force s is a reactionary force, and subject to crushing strength of the material is unlimited, but only so much of it comes into action as is required to counterbalance some other force operating on it in a given direction; m cos α is one such force and the thrust of the wall is another.

The supporting action of the wall is a thrust primarily in direction perpendicular to the face of the wall, and is therefore in this case horizontal. Hence there are now the directions of three forces, and the amount of one of them, mf, or m cos α. So the force parallelogram can be constructed, and by drawing parallels to mf and s respectively, meeting at R, the length gR or R is obtained, and P, the force equal and opposite to R, represents the thrust exerted by the wall.

See then Figure 7. xy is the ground line horizontal, and is parallel to gr, also to fv; yzg is at right angles to xzt, the base of the wedge abt, or bt, of Figure 2; and mr is parallel to gy (by construction, see above), therefore angle xyg = angle grm; xa represents ab or h, of Figure 1, and is perpendicular to xy, and axt is the angle abt $= \beta$, of Figure 1; whilst angle cxy is the angle of repose α of the material.

$$\therefore \text{axy} = 90^\circ, \therefore \text{xyg} = 90 - (\alpha + \gamma) = \beta$$

$$\therefore \text{grm} = 90 - (\alpha + \gamma) = \beta.$$

mf is parallel to lg, and lg represents the vertical mg of Figure 2, and angle gfv $= \alpha$, $\therefore$ gfm $= 90^\circ - \alpha$ sin fgm $= \dfrac{\text{fm} \sin (90 - \alpha)}{\text{mg}}$ but fm = m, or mg, of Figure 2, and mg = m cos α (or the mf, of Figure 2); therefore—

$$\sin \text{fgm} = \frac{\text{m} \sin (90 - \alpha)}{\text{m} \cos \alpha} = \frac{\text{m} \cos \alpha}{\text{m} \cos \alpha} = 1{\cdot}0 = \sin 90^\circ,$$

therefore fgm $= 90^\circ$, and therefore fmg $= 90 - (90 - \alpha) = \alpha$

therefore angle lgm $= \alpha$, but angle lgr $= 90^\circ$,

therefore mgr $= 90^\circ + \alpha$,

$$\text{therefore gmr} = 180 - \{(90 + \alpha) + \beta\} = 90^\circ - (\alpha + \beta)$$

$$\text{gr} = \frac{\text{mg} \sin \text{gmr}}{\sin \text{grm}} = \frac{\text{m} \cos \alpha \sin 90 - (\alpha + \beta)}{\sin \beta}$$

$$= \frac{\text{m} \cos \alpha \cos (\alpha + \beta)}{\sin \beta}$$

but gr = R = P of Figure 2.

Therefore,

$$\text{P} = \frac{\text{m} \cos.\alpha \cos (\alpha + \beta)}{\sin \beta}$$

Figure 4 shows the force polygon for the wedge or triangle.

But it has been shown that—

$$m = \frac{k\,h^2 \cos a \sin \beta}{2 \cos (a + \beta)}$$

$$\text{Therefore } P = \frac{k\,h^2 \cos a \sin \beta}{2 \cos (a + \beta)} \times \frac{\cos a \cos (a + \beta)}{\sin \beta}$$

$$\text{Therefore } P = \frac{k\,h^2 \cos^2 a}{2}$$

as a general equation for embankments.

And since the triangle abt, of Figure 1, is any triangle in the pressure producing portion of the embankment, the pressure on the wall emanating from the surcharged backing is the same whether the angle β approximates O, or whether it is anything between that and $(90 - a)$, the slope of repose; and since the portion of the backing cbd, of Figure 1, produces no effect, the same result is true for the whole embankment.

From this expression is obtained the following results, namely:—If the backing be rock standing perpendicular behind the wall, when obviously there can be no pressure upon the wall, β is O and is 90°, and $\cos^2 90^\circ = 0$, therefore

$$P = \frac{k\,h^2 \cos^2 90^\circ}{2} = 0$$

When $\beta = (90 - a)$, if the mass be a single concrete mass resting at its angle of repose, in which case again, of course, there is no pressure on the wall, then m is or may be, infinite, and at, (Figure 1) is infinite; then no definite value for $\frac{k\,h^2}{2}$ is possible, and we must use the expression

$$P = \frac{m \cos a \cos (a + \beta)}{\sin \beta} \text{ in which } \beta = (90 - a)$$

$$P = \frac{m \cos a \cos \left\{a + (90 - a\right\}}{\sin (90 - a}$$

$$P = \frac{m \cos a \cos 90^\circ}{\cos a} = m \cos 90^\circ = m \times o = 0.$$

implying that with this rigid mass the friction at the angle of repose counterbalances the action of gravity, which obviously it must do.

If β be any other angle between O. and $(90^\circ - a)$, or, if in the last instance m is not a rigid mass, but, as it were, a congregation of loose grains, then m will contain, as it were, loose wedges within it, and then—

$$P = \frac{kh^2 \cos^2 a}{2}$$

representing that with varying values for the angle β, the weight m, and the friction increase in corresponding ratio, so that the maximum is reached with any small plus value of β. provided β is large enough to allow the grains of the material to trickle down and fill the triangle.

If this general expression $P = \frac{kh^2 \cos^2 \alpha}{2}$ is true for any class of solid material, retained in position by a wall, it should also be true for fluids. The angle of repose for fluid is $\alpha = O$, and water at rest assumes an horizontal surface; then $P = \frac{kh^2 \cos^2 \alpha}{2} = \frac{kh^2 \cos {}^2 O}{2} = \frac{kh^2 \times 1^2}{2}$

$P = \frac{kh^2}{2}$ but $k = 62 \cdot 4$ for water. Therefore, $P = \frac{62 \cdot 4\ h^2}{2}$ which is exactly as obtained from an entirely different process of reasoning.* This, therefore, confirms the logic of the whole of the above argument, and the general equation for retaining walls for all classes of material is—

$$P = \frac{kh^2 \cos^2 \alpha}{2}$$

It can be readily proved that the distance, L, of the centre of pressure or locus of the resultant force P, is $\frac{h}{3}$ from the base. Hence $P\ L = \frac{kh^3 \cos^2 \alpha}{6}$, for all classes of backing to retaining walls including water.

To complete the subject of surcharged walls it may be added that in the case of Figure 5, which may be described as truncated surcharge, the same equation holds good, for since the wedge abt alone will provide the maximum pressure, no other area can produce more, and yet the truncated form does not afford less.

BACKING LEVEL WITH CREST OF WALL.

In Figure 6, if bt lies upon ba, obviously, and as before shown, $m = O$, and therefore $P = O$. If bt lies on bc, the mass m is met by the friction m sin α, and with it is converted into m cos 90°, as also shown above, and again, $P = O$. If bt be taken as lying in any position between these extremes, it will be found upon trial that the more nearly bt approaches bc the less is the value of P; whilst the more nearly bt approaches ba the greater is the value of P, until, if, when h is 12 feet, the backing loose sand at 100 pounds per cubic foot and the angle of repose, $\alpha = 33^\circ\ 42'$, and the angle β be taken as one minute of arc, it will be found that $P = 4{,}982$ pounds, or practically the same as $\frac{kh^2 \cos^2 \alpha}{2}$ which, under the

* See equations (supra) for Walls without cohesion retaining liquid.

conditions, equals 4,983·4 for the surcharged wall; the difference, 1·4 pounds, represents the added effect of the triangle ats of Figure 8, when the angle β is one minute of arc. Thus, it is again evident that in all cases the stress is at once brought to its maximum by the layers of sand immediately adjacent to the wall; hence the above equations apply for all cases, and a general statement for embankments of any shape, provided they consist wholly of the one description of material, is—

$$P = \frac{kh^2 \cos^2 \alpha}{2} \qquad L = \frac{h}{3}$$

$$PL = \frac{kh^3 \cos^2 \alpha}{6}$$

Thus, it is evident that all materials supported by a wall are amenable to the same laws, but that friction enters the case with solids to modify the effects. If sand be subjected to heavy drainage, and all its interstices become filled with water, friction and angle of repose disappear, and the mixture tends to assume the horizontal state of rest of a fluid, but of a fluid having specific gravity greater than that of water. Such a mixture must evidently conform absolutely with the hydrostatic law, and if k is the weight of the mixture in pounds per cubic foot, $P = \frac{kh^2}{2}$

CONCLUDING REMARKS

In the above paragraphs, attempt has been made to outline a method upon which the circumstances of each particular case may be examined. No theory can eliminate the demand for knowledge and experience in the designer, and in the case of ordinary retaining walls this is particularly the case. No general equation is possible for such walls, except that of $fPL = Wl$. In that P is frequently impossible to determine with certainty, because so often the elements producing it are themselves indeterminate and depending upon variety of conditions, and often upon improbable but possible contingencies. L may usually be taken as $\frac{h}{3}$ but circumstances may arise when the centre of pressure assumes a higher position, say, for instance, in such a case as a backing of loose sand superimposed upon a base consisting of material of considerably steeper slope of repose. W, the weight, is simple enough if it stands alone, but if the value of c be admitted, judgment and knowledge of the materials and workmanship to be employed are necessary for the assessment of fair and safe value, which certainly should be kept low; it has been shown that great economy can be effected with very low values for this factor. Meanwhile there still appears to be a great field for the investigator, in the determination of the proper ratios of strength of different cohesive substances, for there appears to be singularly little in the way of reliable evidence concerning tensile, adhesive and shearing strength of mortars, particularly the two latter.

The writer has, of course, seen many retaining walls and has built a good many, but believes he is correct in stating that he has never yet seen one that is built either to text-book dimensions or those of the "no cohesion" section of the table of walls given above. He knows several walls in Western Australia that have been supporting loose sand backing, more or less liable to drainage water, that have stood 15 or 20 years to his knowledge, and which do not contain half the text-book dimensions. He has seen one, and only one failure under such circumstances; that was a rectangular wall only 9 inches thick, brick in lime mortar 6 feet high, and with no weepholes at the base. Even that wall stood erect for 21 years, sustaining fine sand backing level with the crest, with rain and drainage on it every wet day. It finally collapsed when a drainpipe burst at the foot of the backing.

The general immunity from disaster of walls theoretically weak on text-book principles, is common experience to many, and is frequently commented upon amongst architects and engineers in Australia. The result is that most of one's clients —in the case of comparatively small walls, anyhow—flout the idea of building to text-book recommendations, and apparently they are justified in so doing. In the instance referred to above, the writer was called in to reconstruct the retaining wall, and, quoting English practice—full hydrostatic pressure had to be contemplated—mentioned the size generally advocated for the case. The proprietor simply laughed at the idea, and said that if the writer would not rebuild a rectangular wall, brick in cement, nine inches thick, any of the contractors would do it for him, and should do so. Eventually he permitted the building of a wall about equivalent to No. 4 of the $c = 3{,}600$ b of the table.

Whilst authorities on the subject advocate these enormous factors of safety, it only remains for architects and engineers, in practice in Australia anyhow, to take the responsibility of striking out a line for themselves without the support of authority and precedent, otherwise to see themselves laid on the shelf whilst the required works are constructed by building contractors and amateurs, who are not troubled by considerations of professional responsibility or reputation.